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CONTENTS

Chapter 1

DC Revision

DC Revision

We will start this section with a chapter of revision of some of the very basic ideas of electrical principles; that is with simple DC circuits. It is important to get the fundamentals clear as this will provide a firm foundation on which further work may be based.

An electric current may be considered to be the movement of electric charge through a conductor. An analogy which is useful in some cases is to compare this movement of charge with the flow of water through a pipe, but as with all analogies it must not be taken too far.

Basic Units

The unit of charge (quantity symbol *Q*) is called the coulomb (unit symbol C). The same rules apply to quantity and unit symbols as were met in the section on mechanics. This unit of charge or quantity of electricity is equivalent but of opposite sign to the charge of $6{\cdot}24 \times 10^{18}$ electrons, this is an exceedingly large number. The reciprocal of this gives the charge on one electron (*e*) which is $-1{\cdot}60 \times 19^{-19}$C.

The coulomb is a small unit of charge and a non-standard unit is sometimes used, particularly when considering batteries, this is the ampere hour Ah. 1Ah = 3600C.

The rate of movement of charge through the conductor i.e. the number of coulombs per second passing a point is called the current (*I*) measured in amperes (A). This rate of flow is not the same as the velocity of movement of charge nor is it the rate of transfer of energy. Comparison with the water flow analogy will make these differences clearer. The current is equivalent to the rate of flow of water in say litres per minute (Coulombs/second). The velocity of individual particles of water (molecules) will depend on the diameter of the pipe.

As an example if a pipe with an internal diameter of 25 mm (approx 1 in) was passing water at a rate of 150 ml/s (approx 2 gals/min) the water would have a velocity of about 300 mm/s (approx 1 ft/s). If the pipe was 50m long it would take 3 mins for water to travel from one end to the other. However, provided the pipe was full, if a tap was turned at one end the flow would start or stop at the other end almost instantly. Thus the energy is transferred very quickly.

If a current of 1A is passed by a conductor of diameter 1 mm then the energy will travel along it at almost the speed of light in a vacuum which is by definition 299 792 458 m/s. The speed of light is represented by the symbol *c*. The electrons however, will travel in the opposite direction to the current at a velocity of approximately 0·1 mm/s, nearly 3 hrs to travel a metre.

The force or pressure which causes the flow of current in a circuit is called the voltage (*V*), potential difference (abbreviated to pd) (*V*) or the electromotive force (emf) (*E*). It is measured in volts (*V*). The volt is actually defined in terms of power. A potential difference of one volt exists between 2 points on a conductor carrying a constant current of one ampere when the power dissipated between the points is one watt.

The above definition gives the relationship between voltage, current and power.

$$\boldsymbol{P = VI}$$

As in mechanics, power is measured in watts and energy (*W*) is measured in joules (J). One joule is one watt second. This is a small standard unit of energy and non-standard units are still used in electrical engineering eg Wh and kWh. 1 kWh = 3·6 MJ.

In a circuit the current is limited by the resistance (*R*) in ohms (Ω). The relationship between voltage, current and resistance is known as Ohm's Law and in symbols is expressed as:

$$\boldsymbol{R = V/I} \quad \textbf{or} \quad \boldsymbol{V = R\,I} \quad \textbf{or} \quad \boldsymbol{I = V/R}$$

A network of resistors and sources of emf may be analysed (i.e. the current in each branch determined) using a number of different methods. In the simplest of cases Ohm's Law and the rules for the addition of resistors in series and parallel are sufficient, but in more complicated cases other methods must be used.

Resistors in series may be added:

$$\mathrm{R_T} = \mathrm{R_1} + \mathrm{R_2} + \mathrm{R_3} + \ldots\ldots$$

For resistors in parallel the rule is that the reciprocals of the resistances are added. For DC circuits **only,** the reciprocal of resistance is called the *conductance,* symbol *G*, measured in siemens (S).

$$G_{\mathrm{T}} = G_1 + G_2 + G_3 + \ldots\ldots$$

This can be written:

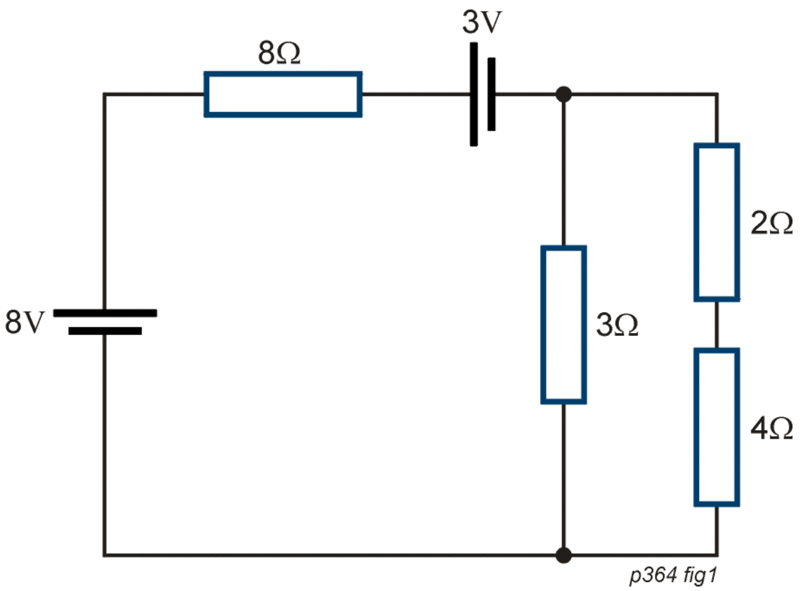
$$\frac{1}{R_{\mathrm{T}}} = \frac{1}{R_2} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\ldots$$

In the case of 2 resistors in parallel this may be further simplified:

$$R_{\mathrm{T}} = \frac{R_1\,R_2}{R_1 + R_2}$$

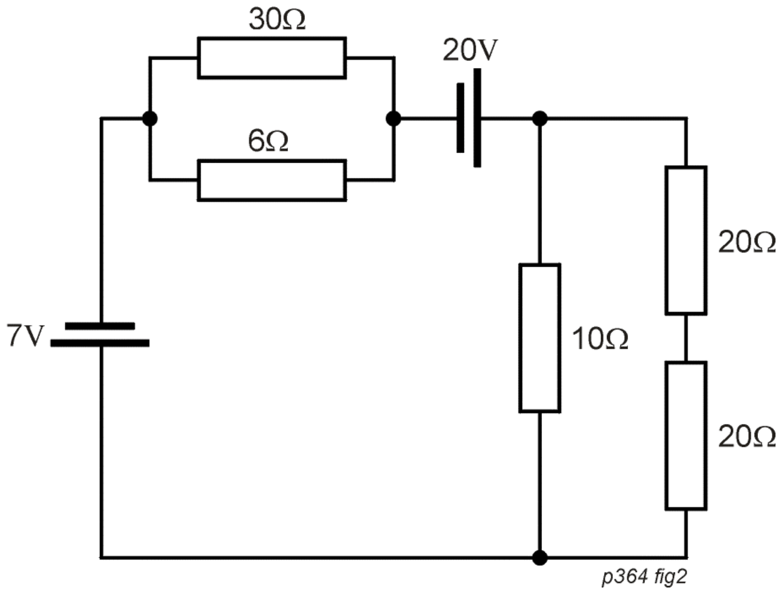
I do not intend to spend any time on the very simple cases but have given you a number of SAQs to use for revision.

SAQ5-1-1 In the circuit shown in the diagram determine the current in each resistor.



SAQ 5-1-2

Determine the current in each resistor of the given network.



SAQ 5-1-3

A circuit, consisting of 3 resistors of 12Ω, 20Ω and 30Ω respectively joined in parallel, is connected in series with a fourth resistor. The whole is supplied at 40V and it is found that the power dissipated in the 12Ω resistor is 48W. Determine the value of the fourth resistor and the total power dissipated in the 3 parallel resistors.

SAQ 5-1-4

Two resistors R_a and R_b are connected in series with each other between the terminals of a battery giving a constant pd of 49V. When a voltmeter of 500Ω resistance is connected across each in turn, the readings given are:

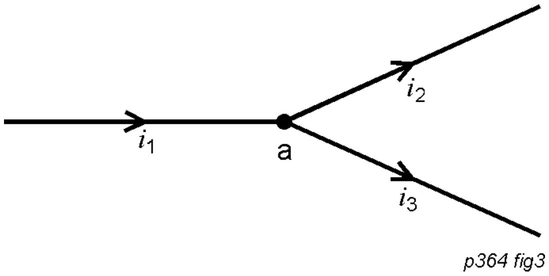
$$V_a = 14\text{V} \qquad V_b = 28\text{V}$$

Determine the resistance of R_a and R_b.

Kirchhoff's Laws

If the circuit is slightly more complicated then other methods of analysis must be employed. These are based on a number of laws and theorems. In a network the junctions where 2 or more wires join are called *nodes* and the wires between nodes are called *branches*.

Since charge cannot accumulate at a node the sum of the currents entering the node must equal the sum of the currents leaving the node. This is known as *Kirchhoff's current law*. The law is more formally stated, for reasons that will become obvious later, as: **at any node in an electrical circuit the algebraic sum of the currents is zero**. The term *'algebraic sum'* means that we have to give a sign to the currents. It doesn't matter whether we take in or out as positive the other direction is then negative. This is shown in the diagram below, where *a* is a node.



$$i_1 - i_2 - i_3 = 0$$

or

$$i_2 + i_3 - i_1 = 0$$

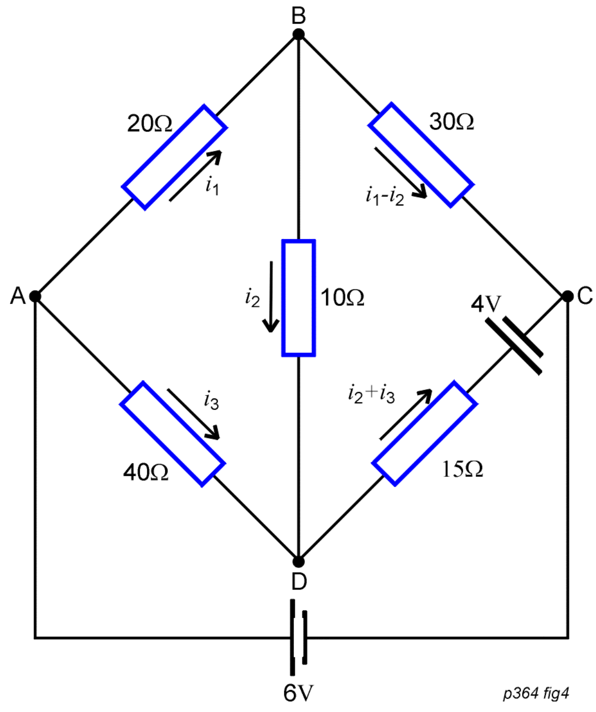
A *closed path* or *loop* is a path that starts at a node and returns to the same node without passing through any intermediate node more than once.

Kirchhoff's voltage law states that: **in any closed path, the algebraic sum of the products of the current and resistance of each part of the circuit is equal to the resultant emf in the circuit.** It is sometimes stated as: the algebraic sum of the pds across the components of a circuit is zero. This can be a useful way of looking at it and although not wrong is not as accurate as the previous definition.

Any network may be analysed by putting unknown currents in each branch in an arbitrary direction and then writing down enough equations using the 2 laws to solve for all the unknowns. If the calculated value for a particular current is negative then it means that the original chosen direction was wrong.

Network Analysis

We will now consider a network and analyse it using Kirchhoff's laws.



In the network shown there are nodes at A, B, C and D. Strictly speaking these are *principal nodes* or *essential nodes*, in accordance with the definition above there is a node between the 15Ω resistor and the 4V battery. I will only be marking essential nodes on diagrams and will just call them nodes.

The method of analysis using Kirchhoff's laws is to put in as many *branch currents* as are necessary, here indicated by black arrows and marked i_1, i_2 and i_3.

Applying Kirchhoff's current law to node B enables us to write down the current in branch BC as $(i_1 - i_2)$ in the direction shown in the diagram by the blue arrows. There is of course a current $(i_1 + i_3)$ flowing through the 6V battery along the branch in the direction CA.

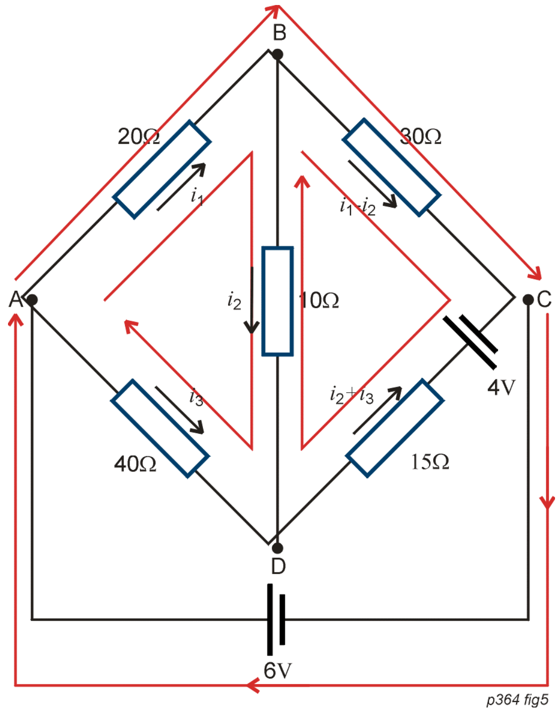
Note that we could have chosen our original 3 currents in any direction in any 3 branches provided they did not all meet at the same node.

To calculate values for 3 unknown currents we will need 3 independent equations. These equations may be obtained by applying Kirchhoff's voltage law to 3 closed paths or loops. If we don't include all branches at least once we will not have an independent set of equations.

In the circuit we have 6 possible loops: ABDA, BCDB, ABCA, ADCA, ABCDA, ABDCA and ADBCA.

We will use the loops ABDA, BCDB and ABCA.

I will show the diagram again below to help you follow how the equations are formed. The loops used have been marked with green lines.



p364 fig5

For loop ABDA

$$20i_1 + 10i_2 - 40i_3 = 0$$

Where the path is in the same direction as the current then the sign in the equation is positive, where the path and the current are in opposite directions then the sign is negative. Note where the sign is positive the path goes from the positive to the negative end of the component. In this loop there are no sources of emf and hence there is no constant term.

For loop BCDB

$$30 (i_1 - i_2) - 15 (i_2 + i_3) - 10i_2 = 4$$

This can be simplified by grouping terms to give:

$$30i_1 - 55i_2 - 15i_3 = 4$$

The left hand side of the equation is the sum of the products of the currents and the resistances. This sum is equal to the sum of the emfs in the loop. In this case the 4V battery is acting in the same direction as the path round the loop and hence is positive as shown on the right hand side of the equation.

If you had been working from the other version of the above law then the +4 volts would appear in the equation as -4 volts on the left hand side. The sign of the voltage goes from negative to positive when moving along the loop in the direction indicated by the green line and arrow heads.

For loop ABCA

$$20i_1 + 30(i_1 - i_2) = 6$$

Grouping terms gives:

$$50i_1 - 30i_2 + 0i_3 = 6$$

We now have 3 equations in 3 unknowns:

$$20i_1 + 10i_2 - 40i_3 = 0$$

$$30i_1 - 55i_2 - 15i_3 = 4$$

$$50i_1 - 30i_2 + 0i_3 = 6$$

These equations may now be solved by a variety of different methods to give the following currents:

$$i_1 = 102{\cdot}8 \text{ mA}, \; i_2 = -28{\cdot}7 \text{ mA} \text{ and } i_3 = 44{\cdot}2 \text{ mA}$$

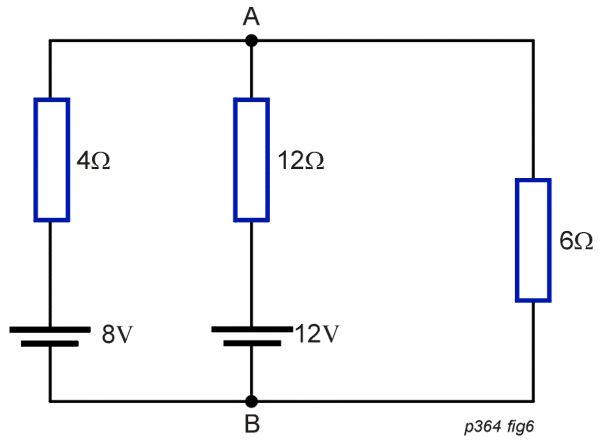
These are the currents in branches AB, BD and AD respectively. As the value of the current in branch BD is negative the current is actually flowing from D to B in that branch. The currents in the remaining branches may now be found by simple arithmetic.

Current from B to C = $(i_1 - i_2) = 102{\cdot}8 + 28{\cdot}7 = 131{\cdot}5$ mA

Current from D to C = $(i_2 + i_3) = -28{\cdot}7 + 44{\cdot}2 = 15{\cdot}5$ mA

Current from C to A = $(i_1 + i_3) = 102{\cdot}8 + 44{\cdot}2 = 147{\cdot}0$ mA

SAQ 5-1-5 Using Kirchhoff's laws determine the current in each of the branches in the network shown. Determine both the magnitude and direction. Don't determine the magnitude and then guess at the direction. It's quite easy to guess the direction in this case but no so easy in the case above.



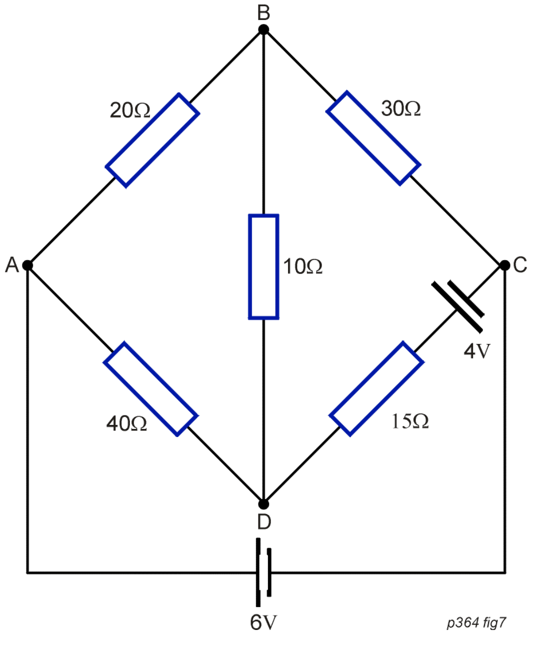
Using this method of approach to the problem is not always easy to see where to put in the original unknown currents and it is my experience that students frequently make mistakes in determining expressions for currents in the remaining branches using Kirchhoff's current law. In view of this a second method of approach, still using Kirchhoff's second law to determine the equations, but defining the currents in a different way is less prone to errors and is recommended. This method uses what are called *Maxwell's circulating currents*. In order to explain this method we must define a *mesh*.

A mesh is a loop which does not enclose any other loop or alternatively it is a loop in which no node is connected to more than 2 other nodes in the same loop.

In the circuit we analysed before there are 9 loops but there are only 3 meshes.

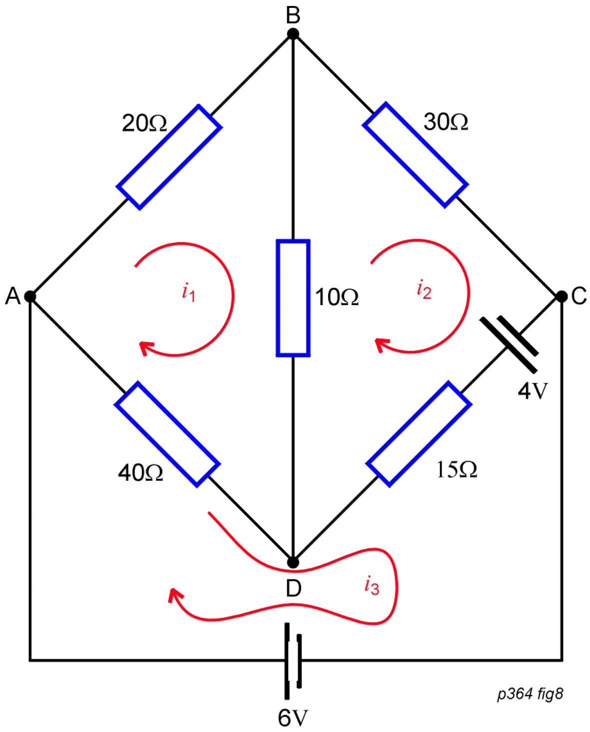
The meshes are:

ABDA
BCDB
ADCA



Maxwell's Circulating Currents

In order to analyse a network using Maxwell's circulating currents a current is inserted in each mesh which is assumed to circulate within that mesh. The currents in all meshes are drawn to circulate in the same direction I have chosen clockwise as the direction of rotation. The network is shown below with the circulating currents marked in green and labelled I_1, I_2 and I_3.



The current in any branch is then the algebraic sum of the mesh currents which flow through that branch. I have used I_1, I_2 and I_3 for the mesh currents to distinguish them from i_1, i_2 etc which I have used for branch currents. There is no rule concerning this but I have found it a practice which works well.

Equations may now be written using Kirchhoff's voltage laws for any loops but it is easiest to write them for the 3 meshes. As it is normal to use these *mesh equations* the method is frequently referred to as *mesh analysis*.

For mesh ABDA

Current in 20Ω resistor $= I_1$

Current in 10Ω resistor $= I_1 - I_2$

Current in 40Ω resistor $= I_1 - I_3$

Therefore applying Kirchhoff's Law:

$$20\,I_1 + 10\,(I_1 - I_2) + 40\,(I_1 - I_3) = 0$$

Hence

$$70\,I_1 - 10\,I_2 - 40\,I_3 = 0$$

Similarly for the other meshes.

For mesh BCDB

$$-10\,I_1 \;+\; 55\,I_2 \;-\; 15\,I_3 \;=\; 4$$

In this case there is a positive 4 on the right hand side as the 4V battery in the circuit is acting in the same direction as the mesh current. The coefficients of the currents are found as before.

For mesh ADCA

$$-40\,I_1 \;-\; 15\,I_2 \;+\; 55\,I_3 \;=\; 2$$

In this case the 2 if from 6 (in the same direction as I_3) minus 4 (in the opposite direction to I_3).

The above lays down a clear procedure to follow even if the whole circuit is much more complicated.

With practice the equations may be written down directly in their final form.

The resulting 3 equations are:

$$70\,I_1 \;-\; 10\,I_2 \;-\; 40\,I_3 \;=\; 0$$

$$-10\,I_1 \;+\; 55\,I_2 \;-\; 15\,I_3 \;=\; 4$$

$$-40\,I_1 \;-\; 15\,I_2 \;+\; 55\,I_3 \;=\; 2$$

These equations may now be solved to give:

$$I_1 \;=\; 102{\cdot}8\text{ mA},\;\; I_2 \;=\; 131{\cdot}5\text{ mA} \;\text{ and }\; I_3 \;=\; 147{\cdot}0\text{ mA}$$

These are the currents in branches AB, BC and CA respectively. The currents in the other branches may easily be determined.

Current from B to D = I_1 - I_2 = 102·8 - 131·5 = -28·7 mA i.e. the current is 28·7 mA from D to B.

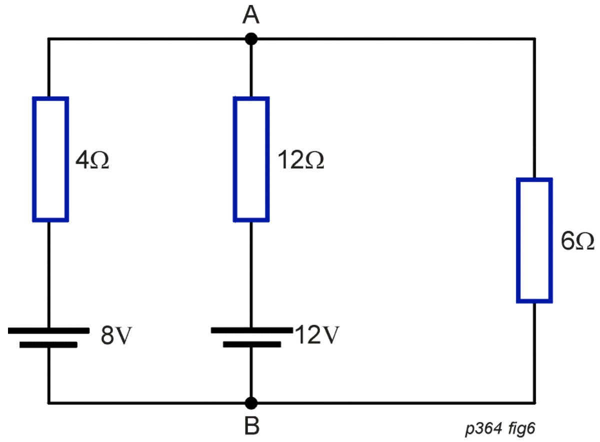
Current from A to D = I_3 - I_1 = 147·0 - 102·8 = 44.2 mA

Current from D to C = I_3 - I_2 = 147·0 - 131·5 = 15·5 mA

You will find that these are the same values as we obtained on page 1-9.

SAQ 5-1-6

Repeat SAQ 5-1-5 but using Maxwell's circulating currents. The diagram is repeated for you.



SAQ 5-1-7

In the network shown in SAQ 5-1-8 below, which of the following paths are loops, which are meshes, and which are neither.

i. ACB

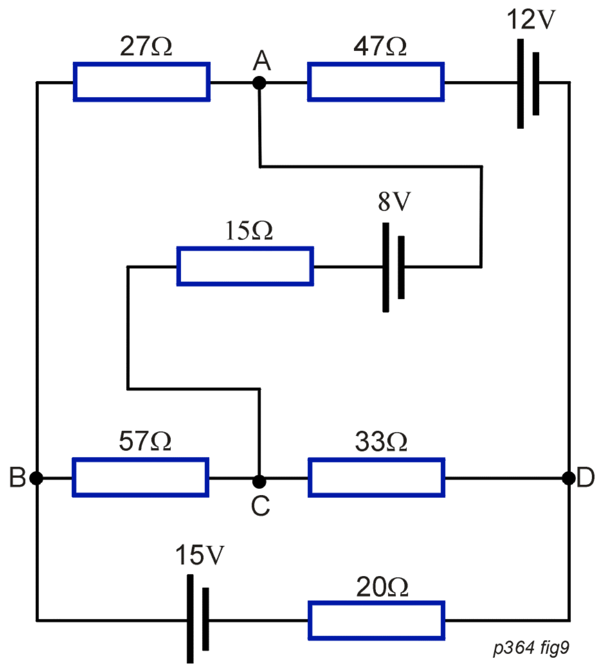
ii. ACBDA

iii. BCDB

iv. ADCA

SAQ 5-1-8

Write down the mesh equations to solve the network below. I am not asking you to solve the equations but in case you decide to do so I have given the values for the circulating currents in the answers.



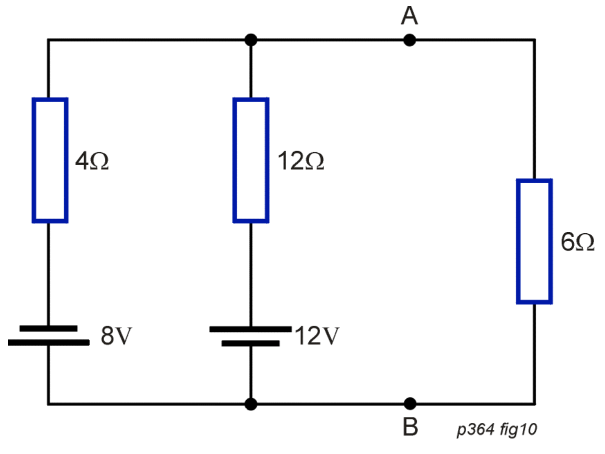
Thévenin's Theorem

Mesh analysis will enable all planar networks (i.e. networks which can be drawn on a flat sheet of paper with no cross-overs) to be analysed and with great care may be used for some non-planar networks. However if the current is only required to be determined in one branch of the circuit less work may be required if Thévenin's Theorem is used. Most students find some difficulty with this theorem but it is very important and will be widely used on your later course at The Royal School of Signals. It not only determines the required current but also when you become familiar with it gives a better understanding of the operation of the circuit.

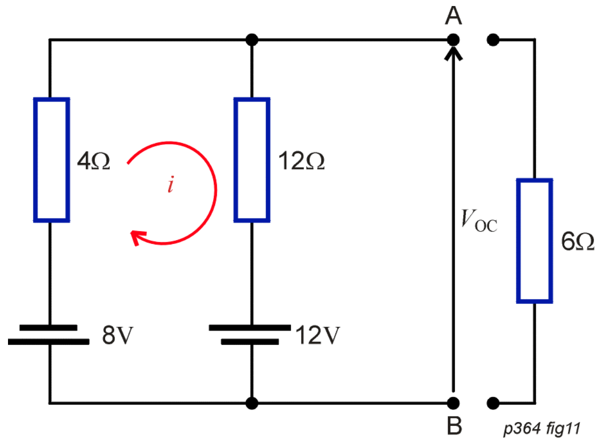
The basic idea of Thévenin's Theorem is that it enables a circuit containing sources of power to be simplified in much the same way that combing 2 resistors in parallel simplifies a circuit. As with most theorems it may be stated in a number of similar but different ways depending on how many possible variations of conditions you wish to cover. One simple statement of Thévenin's Theorem is: **any active, linear, two-terminal network may be replaced by a constant voltage source emf equal to the open circuit voltage at the terminals, in series with a resistance equal to the resistance seen at the terminals with all internal emfs replaced with short circuits.**

That definition probably seems quite a mouth full. Some of the terms need a bit more explanation. **Active** simply means containing a source of power. **Linear** means that every component in the circuit obeys Ohm's law. The **two-terminal network** may be, and in most cases is, part of a larger network.

As an example we will look at the problem you have already tackled in SAQ 5-1-5 and SAQ 5-1-6. The problem is to determine the current in the 6Ω resistor.
I have slightly altered the diagram in that I have indicated 2 points A and B. The part of the circuit to the left of these 2 points or terminals is a 2 terminal active network which may be simplified using Thévenin's Theorem. The 6Ω resistor is connected across the terminals, A and B, of the network we are going to simplify.



In the diagram below I have temporarily removed the 6Ω resistor so that we can find the open-circuit voltage, V_{OC}, of the remaining network. To do this consider the mesh current *I*.



The mesh current is given by:

$$I = \frac{-8-12}{12+4} = -1¼\,\text{A}$$

V_{OC} may now be determined by considering the voltages across either of the 2 branches between A and B, bearing in mind the polarity of the voltages across the individual components i.e. when moving with current, subtract (current × resistance).

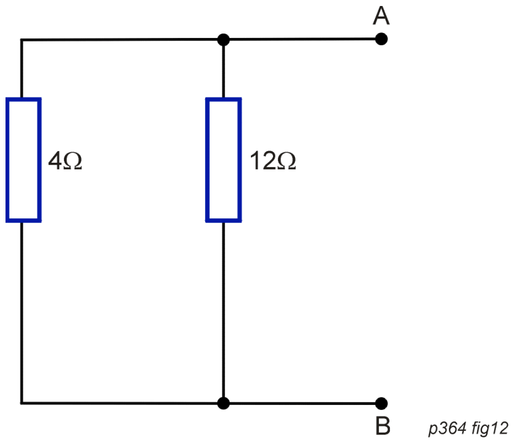
From the left hand branch:

$$V_{OC} = -8 - \left(-1¼ \times 4\right) = -3\,\text{V}$$

From the right hand branch:

$$V_{OC} = 12 + \left(-1¼ \times 12\right) = -3\,\text{V}$$

If we now replace the sources of emf by short circuits we get the circuit below:

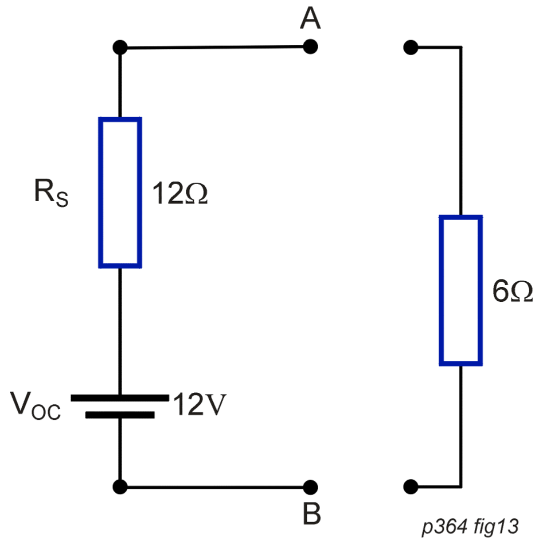


The resistance of this circuit between terminals A and B is given by:

$$R = \frac{4 \times 12}{4+12} = 3\Omega$$

This resistance is in series with the source of emf we have determined above.

The equivalent circuit now becomes:



p364 fig13

I have labelled the source of emf V_{OC} as it is the open circuit voltage of the original two-terminal network. Some people call it V_{th} standing for Thévenin voltage. Note I have drawn the source with the positive terminal towards the A and then marked it –3V.

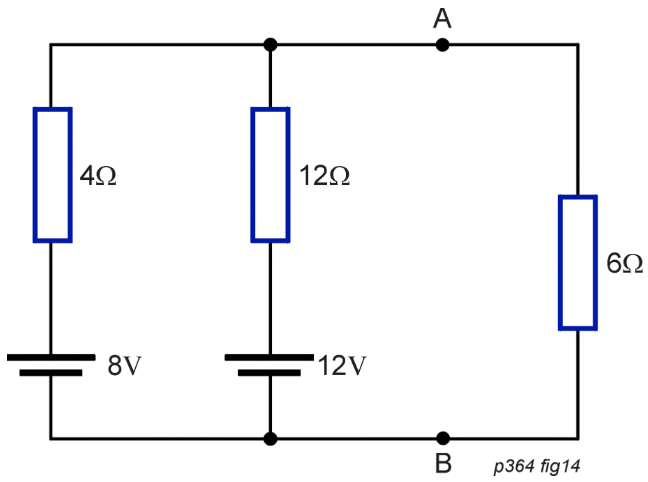
The resistance has been marked R_S which standard for source resistance, it is also known as Thévenin resistance, output resistance or internal resistance.

The circuit has now been simplified to a single loop and the required current in the 6Ω resistor may easily be determined as one third of an ampere from B to A.

It may seem that we have done much more work than is necessary but this is because I have tried to explain clearly what is happening at each stage.

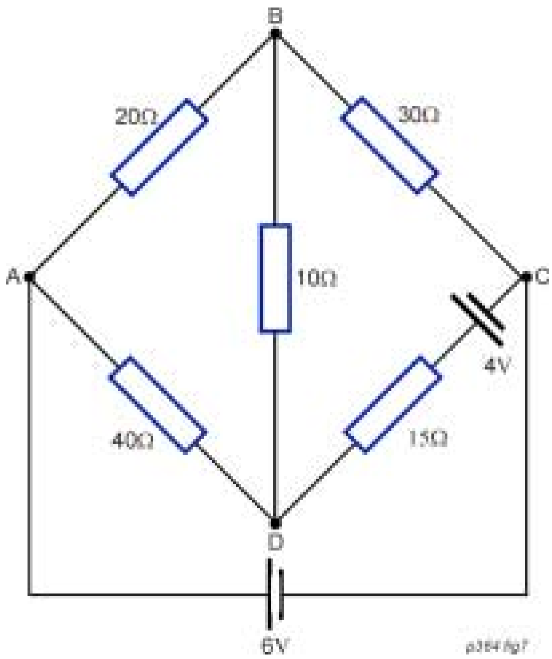
SAQ 5-1-9

In the circuit shown below determine the current in the 6Ω resistor using Thévenin's theorem.



p364 fig14

We will now apply Thévenin's theorem to a more complicated network. We will use the one that we have already analysed using mesh equations.

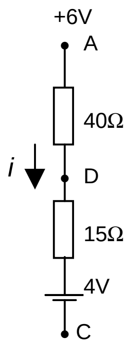


In this case we will just determine the current in the 10Ω resistor in the branch BD. To find V_{OC} for the rest of the network, remove the 10Ω resistor.

B and D are now the terminals of the two-terminal active network comprising all the circuit except the 10Ω resistor.

If we consider there to be an earth at point C, then A is at 6V and the potential at point B may be determined treating the 20 and 30Ω resistors as a potential divider network.

$$V_B = 6 \times \frac{30}{20+30} = 3\cdot 6\,\text{V}$$



To determine the potential at D we need to look at the part of the circuit shown on the left.

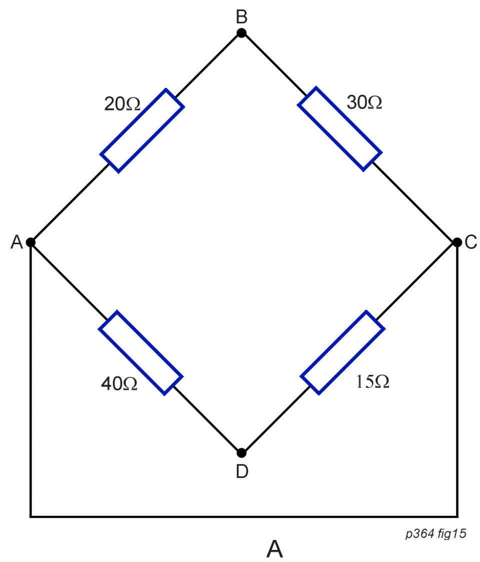
The current $i = \dfrac{6-4}{40+15} = 36\cdot 36\,\text{mA}$

The potential at D is then given by:

$$V_D = 4 + 15 \times 0{\cdot}03636 = 4{\cdot}545\text{V}$$

Then $V_{OC} = V_{BD} = 3{\cdot}6 - 4{\cdot}545 = -0{\cdot}945\text{V}$

In order to determine the source resistance we must look at the network with the sources of emf replaced with short circuits as shown below.

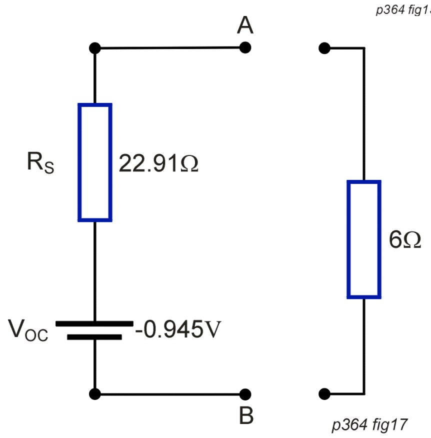


The 2 points A and C are connected together so the total resistance from B to D is the 20Ω resistor in parallel with the 30Ω resistor in series with the parallel combination of the 40Ω and 15Ω resistors.

$$R_S = \frac{20 \times 30}{20 + 30} + \frac{40 \times 15}{40 + 15}$$

$$R_S = 12 + 10{\cdot}91 = 22{\cdot}91\Omega$$

The whole circuit now becomes:



This now enables us to determine the current from B to D as:

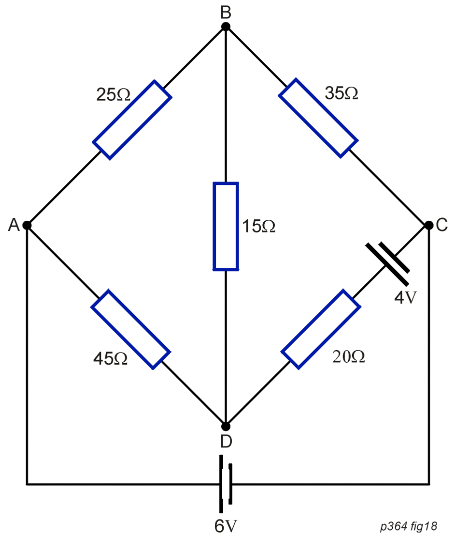
$$i = \frac{-0 \cdot 945}{22 \cdot 91 + 10} = -0 \cdot 0287\,\text{A}$$

Hence current is 28·7 mA from D to B as before.

Thévenin's theorem is very widely used and frequently saves time in analysing a network. It also gives a better understanding than using a computer package to analyse the network. In some cases of course it may involve much more work than other methods and it is a matter of experience to decide which method to use. For example if there were a resistor in series with the 6V battery in the example we have just done it would make it very much more difficult and would almost certainly be easier using mesh analysis.

SAQ 5-1-10

Here is a similar problem with changed values for you to work out the current in the 15Ω resistor.



That completes the chapter on DC and will now move on to deal with basic AC.

Chapter 2

AC Basics

AC Basics

DC is important from the point of view of biasing arrangements and power supplies but AC is very much more important as all signals have AC components. The graph of the variation of voltage or current with time is called its *waveform* and the waveform may be displayed on a cathode ray oscilloscope.

A *periodic* waveform is one in which the shape is regularly repeated after a set constant period of time, called the *period (T)*. An AC waveform is a periodic waveform where the mean value over a complete period is zero. The pattern which repeats is called a *cycle*. The number of cycles per second is called the *frequency (f)* which is measured in hertz (Hz). The frequency is the reciprocal of the period.

$$f = 1/T$$

Sine Waves

The simplest waveform to handle from a mathematical point of view is a sine wave. It can also be shown that any more complicated waveform may be broken down into the sum of a DC value and a number of sine waves. The mathematical expression for a sine wave of *instantaneous* value *v* includes the *peak* or maximum values $\left(\hat{V}\right)$ and the frequency.

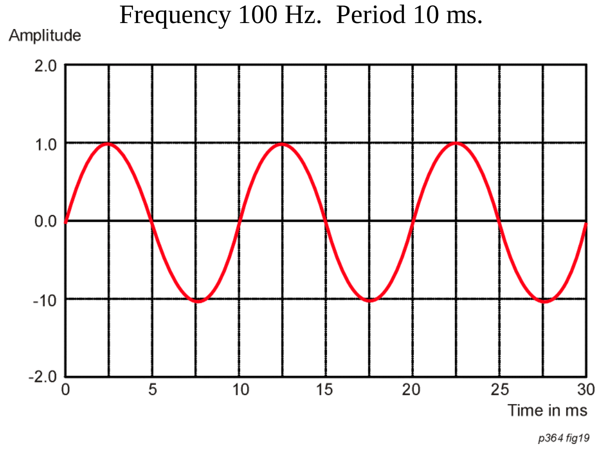
$$v = \hat{V} \sin 2\pi\, ft$$

Where *t* is the instantaneous time.

$2\pi f$ is called the angular frequency which is measured in radians per second and represented by lower case omega (ω).

$$v = \hat{V} \sin \omega t$$

Frequency 100 Hz. Period 10 ms.



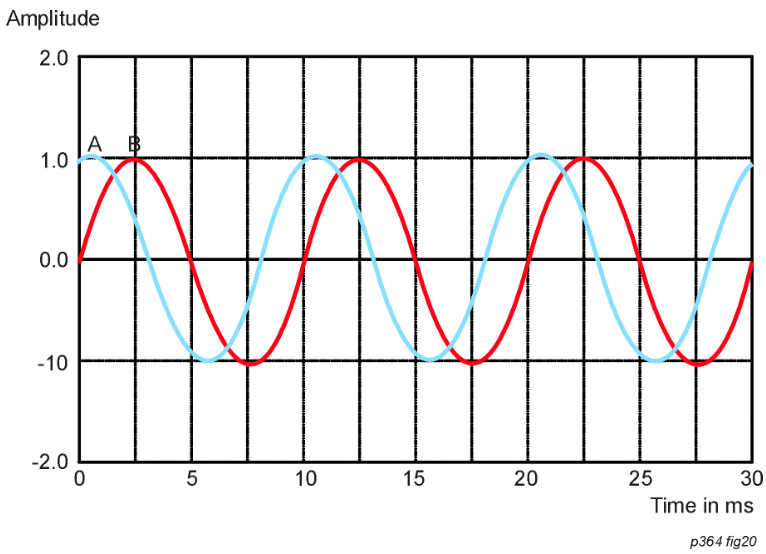
The horizontal scale is time but it could have been marked off in degrees, or in radians, one cycle, which takes 10 ms, being 360° or 2π radians.

Phase Difference

The above expression is for a sine wave for which v = 0 when t = 0. If the sine wave starts at an earlier time which may be represented by an angle then the mathematical expression becomes:

$$v = \hat{V} \sin(2\pi ft + \phi)$$

WAVES WITH DIFFERENT START TIMES



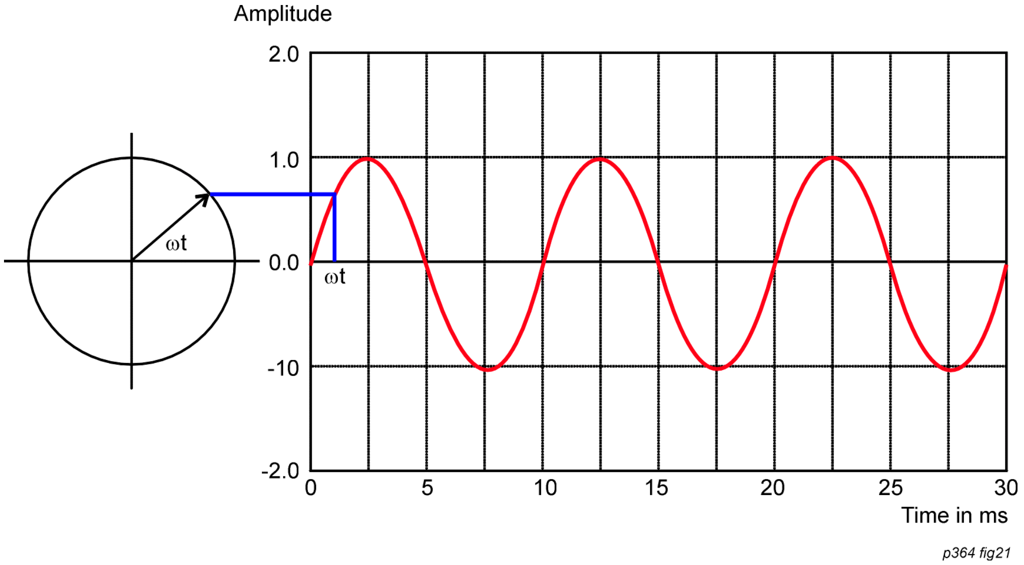
Both the waveforms shown have the same period of 10 ms and the same frequency of 100 Hz. Waveform B starts at value 0 at time t = 0.
Waveform A starts about 2 ms earlier and is said to lead the waveform B. Waveform B lags waveform A, it gets to the same value later. Both these waveforms have the same maximum value but that is not necessary for them to have a phase difference. They must however have the same frequency for phase difference to have any meaning.

The phase difference is normally expressed as an angle either in degrees or in radians, in the example it is about 60°. It is sometimes expressed as a fraction of a cycle. If you use ωt or $2\pi ft$, ϕ should be in radians but provided you remember what you are doing it is frequently easier to use degrees.

Phasor Representation

A sine wave may be represented by a line, called a phasor, of length proportional to the amplitude of the sine wave and rotating in an anti-clockwise direction at the same number of revolutions per second as the frequency of the sine wave in hertz. Some authors have called these lines sinors or complexors and even vectors, but I will use the now standard term phasor. The instantaneous value of the sine wave is then given by the projection of this phasor onto the vertical. Two sine waves of different start times but of the same frequency may then be represented by 2 phasors with the phase angle between them.

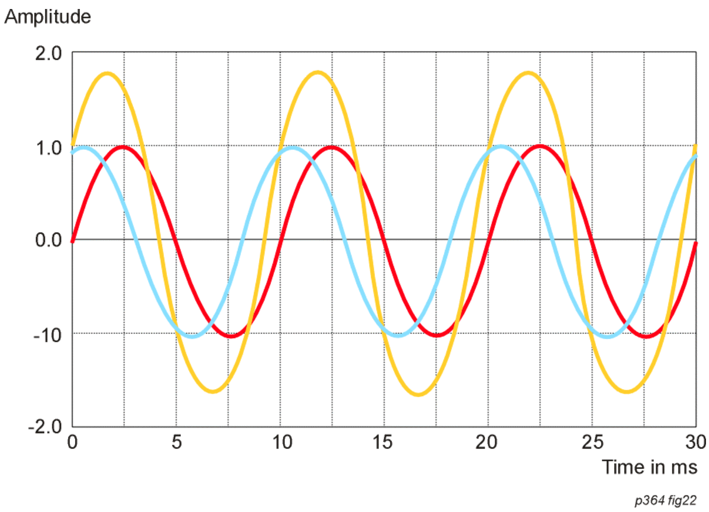
The diagram shows the relationship between a phasor and the sine wave it may be assumed to generate.



If the alternating quantity is a sine wave of voltage then an open arrow head (→) is placed at the end of the phasor which rotates. For currents a solid arrow head (⟶▶) is used.

Sum of Two Sine Waves

The sum of 2 sine waves of the same frequency is represented by the phasor which is obtained by adding together the phasors representing the individual sine waves using the rules of vector addition. This may be done graphically, by trigonometry, or by representing the phasors as complex numbers.



The diagram shows 3 sinusoidal waveforms; sin ωt - [B], sin (ωt + 60°) - [A]. Adding [A] and [B] gives waveform [C], 1·732 sin (ωt + 30°). [C] could be drawn by adding the magnitudes of the other 2 graphs for each point.

From trigonometry:

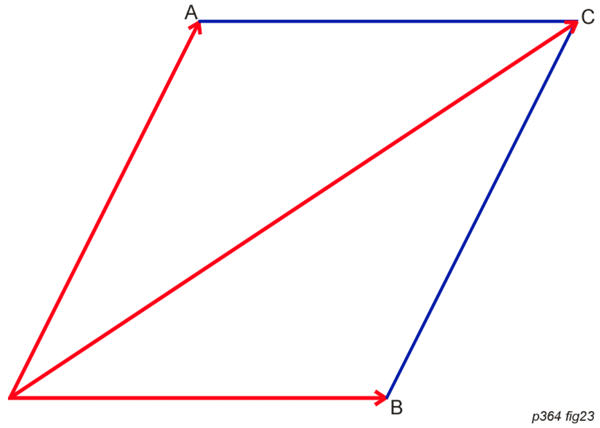
$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

Putting in the values in this case gives:

$$\sin \omega t \;+\; \sin(\omega t + 60°) = 2 \sin(\omega t + 30°) \cos - 30°$$

$$= 1°732 \sin(\omega t + 30°)$$

Using phasors the sum may be determined by drawing phasors to represent waveforms A and B and then using the laws of vector addition to determine the sum, waveform C.



Using complex arithmetic, the waveform B is $1\angle 0° = 1 + j\,0$, and waveform A is represented by $1\angle 60° = 0{\cdot}5 + j\,0{\cdot}866$.

The sum [C] is therefore $1{\cdot}5 + j\,0{\cdot}966 = 1{\cdot}732\angle 30°$.

In most cases the use of complex arithmetic is the simplest method.

Power in AC Circuits

The power at any instant is the product of the instantaneous voltage *v* and the instantaneous current *i*. The average power which in most cases is much more important is hence the average value of the product of the 2 waveforms over one cycle.

$$v = \hat{V} \sin \omega t$$

$$i = \hat{I} \sin(\omega t - \phi)$$

$$vi = \hat{V}\hat{I} \sin \omega t \sin(\omega t - \phi) = \frac{\hat{V}\hat{I}}{2}\left[\cos\phi - \cos(2\omega t - \phi)\right]$$

Over one complete cycle the average value of $\cos(2\omega t - \phi)$ is zero, hence the average power is

$$P = \frac{\hat{V}\hat{I}}{2}\cos\phi$$

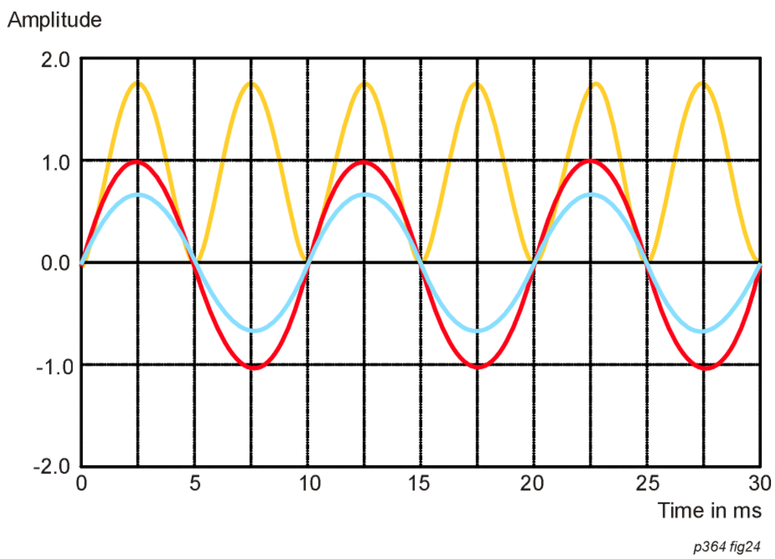
or

$$\boxed{P = VI\cos\phi}$$

where *V* and *I* are the rms values of the AC voltage and current. For a sine wave the rms values are equal to the peak value divided by the square root of 2, which is 1·414. The reciprocal of 1·414 is 0·707. *VI* is called the apparent power (*S*) and cosϕ the power factor. A power supply company requires its consumers to keep their power factor as near to unity as possible.

The graph below shows a voltage waveform of peak value 1V with an in phase current waveform of 0·6A. The graph of instantaneous power is shown at an enlarged scale. Its peak value is 1 × 0·6 = 0·6W but is shown at over twice this value for clarity. It is a cosine wave and where it touches the axis it is curved. It also has twice the frequency of the voltage.
An AC voltage has a peak value of 5V and a frequency of 4 kHz.

POWER - VOLTAGE AND CURRENT IN PHASE



v voltage   i current   p instantaneous power

not to scale

SAQ 5-2-1

i. What is the period of the waveform?

ii. What is the rms value of the voltage?

iii. If at time *t* = 0 the instantaneous voltage is 2V and the voltage is decreasing, what is the phase angle of the waveform relative to a wave starting at *t* = 0? Give the mathematical expression for the wave.

SAQ 5-2-2

Two sine waves both have peak amplitude 7V and frequency 5 kHz. One starts at time t = 0 and the second one leads the first with a phase difference of 30°. Determine the instantaneous voltage of each wave at time t = 25 μs.

SAQ 5-2-3

Use complex numbers to find the sum of the 3 waveforms given below expressing the result in the form $\hat{V} \sin(\omega t + \phi)$

$v_a = 200 \sin \omega t$ $\quad v_b = 170 \sin (\omega t - \pi/4)$ $\quad v_c = 230 \sin (\omega t + \pi/6)$

Chapter 3

Impedance

Impedance

If an AC voltage is applied to a circuit, as in the DC case a current will flow, but in this case both the **magnitude** of the current and its **phase** relationship to the voltage must be determined.

The ratio of the voltage to the current, both of which may be represented by complex numbers, is a complex quantity called the *impedance* (*Z*). The impedance is a complex operator. Multiplying it by current (complex) will give the voltage (also complex) with magnitude and phase different from those of the current. The voltage and current may be represented by phasors but to add a voltage to a current is meaningless. The complex value of impedance may be represented by a point on an argand diagram. The line joining the origin to the point is sometimes called a complexor. Complexors obey the rules of vector addition but are not vectors in the mechanics sense.

AC Ohms Law

The equivalent of Ohms law in the AC case is:

$$V + I\,Z$$

Where *V, I* and *Z* are all complex numbers. Note that *X* may be positive or negative.

$$Z + R + \mathrm{j}\,X = |Z|\angle\phi$$

Where *R*, the real part of the impedance is called the *AC resistance*. X, the imaginary part of the impedance is called the *reactance*.

Z, R and *X* are all measured in ohms.

The *modulus of the impedance* $|Z| = \sqrt{}\,(R^2 + X^2)$

The *angle* of the impedance = arctan *X/R* = arctan *Q*

Q is a symbol used for the ratio *X/R*.

Components

A pure resistor has an AC resistance *R* equal to its DC resistance. In practice resistors have some inductance and the AC resistance at higher frequencies will be higher than the DC value.

A pure inductor will have no resistance and will have reactance given by:

$$X = \omega L = 2\pi f L$$

Where *L* is the inductance in henries (H).

A practical inductor is a coil of wire which may be wound on a core of magnetic material. The coil will have an AC resistance which will be greater than the DC resistance. The reasons why it is greater need not concern us at the moment but it is important to realise that all practical inductors have resistance.

The impedance of an inductor or coil is

$$Z = R + j\omega L$$

SAQ 5-3-1

A coil has a resistance of 14Ω which may be considered to be constant at all frequencies and an inductance of 600 μH. determine its reactance and impedance in rectangular and polar form at frequencies of 2 kHz, 10 kHz and 100 krad/s. If a voltage of 10V at each of the above frequencies is applied across the coil what will be the current in each case?

A capacitor of capacitance *C* farads (F) has a reactance given by:

$$X = -1/\omega C = -1/2\pi f C$$

Note the reactance of a capacitor is negative. Practical capacitors have a very small series loss resistance which may be ignored in most cases.

The impedance of a capacitor is:

$$Z = j\ 1/\omega C$$

SAQ 5-3-2

Determine the reactance and impedance of a 100 nF capacitor at frequencies of 10 kHz and 250 kHz.

Impedances in series

If 2 impedances Z_1 and Z_2 are in series, the total impedance is the sum of the 2 impedances.

$$Z = Z_1 + Z_2$$

This rule may obviously be extended to any number of impedances in series.

Impedances in parallel

If 2 impedances Z_1 and Z_2 are in parallel, the total impedance is given by the product divided by the sum of the 2 impedances.

$$Z = \frac{Z_1 \times Z_2}{Z_1 + Z_2}$$

The above 2 expressions are of course the same as the expressions we had for resistors in series and parallel, which is as we would expect, as a resistance is an impedance with only a real part.

Admittance

When we were dealing with resistors in parallel I introduced the term conductance being in that case the reciprocal of the resistance. In the case of impedance the reciprocal of impedance is *admittance*. The symbol for admittance is Y and as with conductance it is measured in siemens (S).

In the same way that impedance has real and imaginary parts called resistance and reactance respectively so admittance has real and imaginary parts called conductance (G) and susceptance (B).

$$Y = G + \mathrm{j}B$$

G and B may be determined in terms of R and X. The relationship between G and R which was given before for the DC case is **NOT** true in the AC case.

$$Y = \frac{1}{Z} = \frac{1}{R + \mathrm{j}X}$$

If we rationalise this expression by multiplying the numerator and the denominator by the complex conjugate we obtain:

$$Y = \frac{1}{R + \mathrm{j}X} \times \frac{R - \mathrm{j}X}{R - \mathrm{j}X}$$

$$Y = \frac{R}{R^2 + X^2} - \frac{\mathrm{j}X}{R^2 + X^2}$$

But $\quad Y = G + \mathrm{j}B$

$\therefore$ $$G = \frac{R}{R^2 + X^2} = \frac{R}{|Z|^2}$$

and $$B = \frac{-X}{R^2 + X^2} = \frac{-X}{|Z|^2}$$

These expressions may be written in alternative forms but the above will be adequate for the moment.

SAQ 5-3-3

What is the total impedance of $47\angle 35°\Omega$ and $32 + j\ 17\ \Omega$

a. in series

b. in parallel

in both polar and rectangular form?

SAQ 5-3-4

What is the admittance, conductance and susceptance of an impedance of $15 - j\ 20\ \Omega$?

SAQ 5-3-5

Electrical measurements are made on a component at a frequency of 25 kHz and its impedance is found to have a magnitude of 85Ω and a resistive component of 15Ω. If it is an inductor determine its inductance and if it is a capacitive circuit determine its capacitance. Explain why it is impossible to determine from the information given which circuit it is.

SAQ 5-3-6

A circuit is found to have a conductance of 30 mS and a reactance of +16Ω. Determine the 2 possible values for both the resistance and the susceptance.

AC Network Analysis

The various laws and theorems which we have used to analyse DC networks may be used to analyse AC networks but we must now remember that all our voltages and currents are represented by complex numbers. The resistors are now impedances.

Kirchhoff's current law

At any node in an electrical circuit the complex sum of the currents is zero.

Kirchhoff's voltage law

In any closed path, the complex sum of the products of the current and impedance of each part of the circuit is equal to the complex sum of the emfs in the circuit.

Thévenin's Theorem

Any active, linear, two-terminal network may be replaced by a constant voltage source of emf equal to the open circuit voltage at the terminals, in series with an impedance equal to the impedance seen at the terminals with all internal emfs replaced with short circuits.

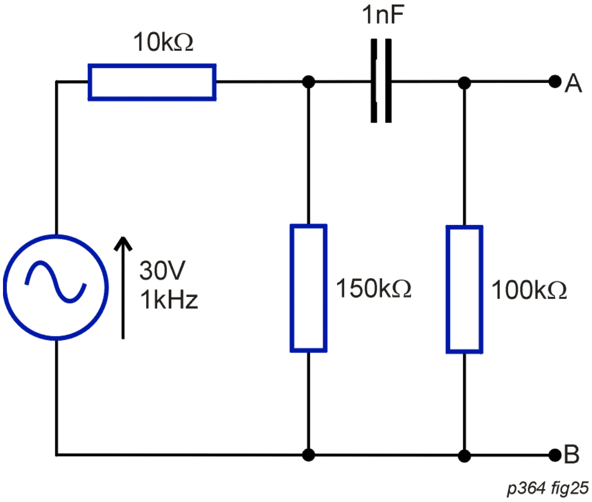
Maxwell's circulating currents may of course be used in AC circuits.

As in the DC case the best way to learn about these is to go through some worked examples.

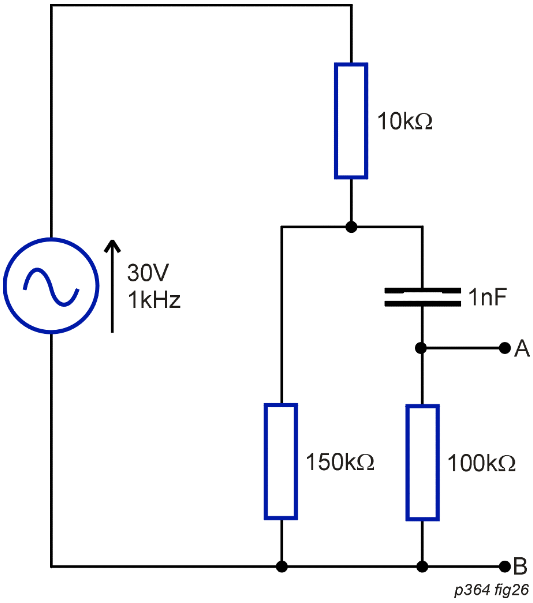
AC Worked Example

We will start by looking at a simple network reducible to one mesh.

In the circuit below determine the output voltage, between terminals A and B, in magnitude and phase relative to the source.



Redrawing the circuit shows more clearly that it is a resistor in series with a parallel combination.



In order to determine the impedance of the circuit we must first determine the reactance of the capacitor.

$$X_C = \frac{-1}{\omega C} = \frac{-1}{2 \times \pi \times 1 \times 10^3 \times 1 \times 10^{-9}}$$

$$X_C = -159 \text{ k}\Omega$$

The parallel part of the circuit is 150 kΩ in parallel with 100 - j 159 kΩ.

The total impedance of this part of the circuit may now be determined by calculating the product divided by the sum. I will assume that you have calculators capable of conversion from rectangular to polar and vice-versa. We will work in volts and kilohms which will give us currents in milliamperes.

$$Z_p = \frac{150 \times (100 - \mathrm{j}159)}{150 + (100 - \mathrm{j}159)}$$

The denominator is 250 - j 159 = 296·3∠-32·5°

$$\therefore \qquad Z_p = \frac{150 \times 187 \cdot 8\angle -57 \cdot 8°}{296 \cdot 3\angle -32 \cdot 5°}$$

$$Z_p = 95 \cdot 07\angle -25 \cdot 3° = 85 \cdot 95 - \mathrm{j}40 \cdot 63 \,\mathrm{k}\,\Omega$$

The total impedance of the circuit is the above plus 10 kΩ.

$$Z = 95{\cdot}95 - \mathrm{j}\ 40{\cdot}63 = 104{\cdot}2\angle -22{\cdot}95°$$

The total current in the circuit is given by:

$$I = \frac{V}{Z} = \frac{30}{104 \cdot 2\angle -22 \cdot 95°}$$

$$I = 0{\cdot}288\angle 22{\cdot}95° \text{ mA}$$

This current flows through the parallel part of the circuit and hence the voltage across the parallel circuit V_p is given by:

$$V_p = I \times Z_p = 0{\cdot}288\angle 22{\cdot}95° \times 95{\cdot}07\angle -25{\cdot}3°$$

$$V_p \quad 27{\cdot}38\angle -2{\cdot}35° \text{ V}$$

Note that as the impedance of the parallel part of the circuit is much larger than the 10 kΩ series resistor, V_p is only slightly smaller than the applied voltage and with only a small phase change.

The current through the capacitor and 100 kΩ resistor is given by:

$$I_c = \frac{27 \cdot 38\angle -2 \cdot 35°}{187{\cdot}8\angle -57{\cdot}8°}$$

This is V_p divided by 100 - j 159 in polar form which was calculated above.

$$I_c = 0{\cdot}146\angle 55{\cdot}45° \text{ mA}$$

The voltage between A and B, V_{AB} is given by this current times the 100 kΩ of the resistance.

$$V_{AB} = 14{\cdot}6\angle 55{\cdot}45° \text{ V}$$

The voltage across the output of the circuit is 14·6V and leads the voltage source by 55·45°. In most cases if a circuit comprises resistors and capacitors and the output voltage is taken across a resistor it will lead the input voltage and if it is taken across a capacitor it will lag.

That example covered all that is necessary to analyse and solve problems involving circuits reducible to one mesh.

SAQ 5-3-7

A 100V, 60W lamp (pure resistance) is to be operated on a 240V 50 Hz mains supply. Find the values of:

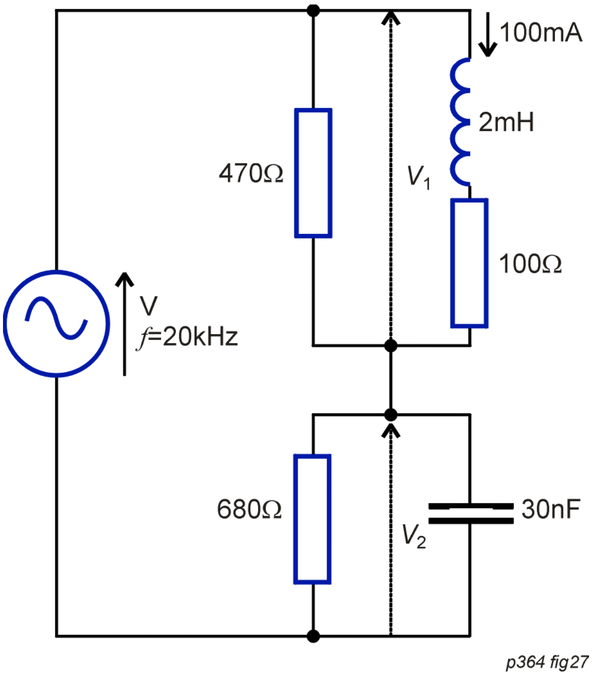
a. Non-inductive resistance

b. Non-resistive inductance

which would be required in series with the lamp to ensure its correct operation and determine the power drawn from the supply in each case.

SAQ 5-3-8

In the circuit below there is a current of 100 mA in the 2 mH inductor. Taking this current as the reference phase determine the current in each of the other components and the 3 voltages V, V_1 and V_2.



That was quite a long problem. I will now give you a couple of SAQs for which I will give you the answers but not the working. They are quite simple but the practice will do you good.

SAQ 5-3-9

A circuit consists of a resistor *R* in series with an inductor *L* and a capacitor of capacitance 0·1 μF. The current through the circuit is 345 mA and the voltages across the components are V_R = 25V, V_L = 40V and V_C = 55V. Determine:

a. the resistance *R*,

b. the frequency,

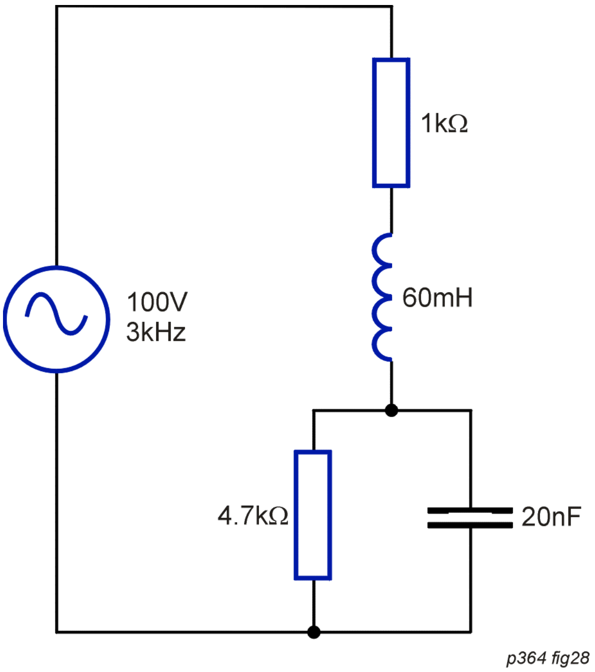
c. the supply voltage,

d. the inductance *L* and

e. the voltage across the inductor and capacitor together.

SAQ 5-3-10

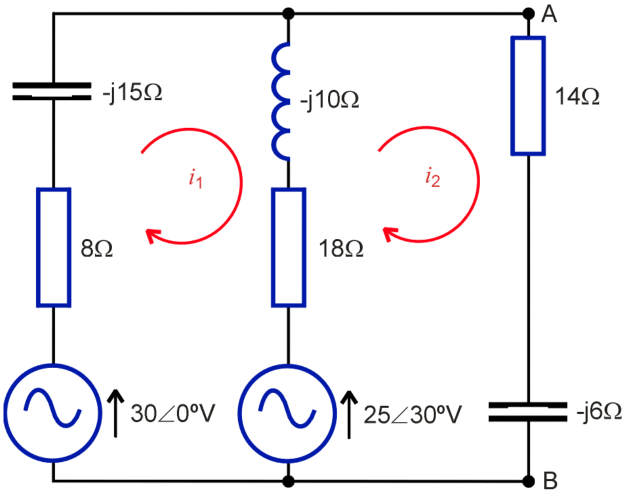
In the circuit shown determine the total current and the current in the capacitor in polar form. Also determine the total power drawn from the supply.



Maxwell and Thévenin's Theorem in AC circuits

We will now use first Maxwell's circulating currents and then Thévenin's Theorem to solve a 2 mesh circuit.

In the circuit shown below determine the current in the branch AB.



p364 fig29

Converting the voltage sources to rectangular form gives:

$$30\angle 0° = 30 + j\,0 \qquad\qquad 25\angle 30° = 21{\cdot}65 + j\,12{\cdot}5$$

We can now write down the 2 mesh equations:

$$(26 - j\,5)\,I_1 - (18 + j\,10)\,I_2 = 30 - (21{\cdot}65 \;\; j\,12{\cdot}5)$$

$$\therefore \quad (26 - j\,5)\,I_1 - (18 + j\,10)\,I_2 = 8{\cdot}35 - j\,12{\cdot}5 \,.\,.\,.\,.\,.\,.\,.\,.\,. \qquad (1)$$

$$-(18 + j\,10)\,I_1 + (32 + j\,4)\,I_2 = 21{\cdot}65 + j\,12{\cdot}5 \,.\,.\,.\,.\,.\,.\,.\,.\,. \qquad (2)$$

Multiplying equation (1) by (18 + j 10) and equation (2) by (26 – j 5) gives:

$$(18 + j\,10)(26 - j\,5)\,I_1 - (224 + j\,360)\,I_2 = 275{\cdot}3 - j\,141{\cdot}5$$

and $\quad -(18 + j10)(26 - j\,5)\,I_1 + (852 - j\,56)\,I_2 = 625{\cdot}4 + j\,216{\cdot}75$

Adding gives: $\quad (628 - j\,416)\,I_2 = 900{\cdot}7 + j\,75{\cdot}25$

$$I_2 = 0{\cdot}941 + j\,0{\cdot}743$$

$$I_2 = 1{\cdot}2\angle 38{\cdot}3°\text{ A}$$

That has determined the current using circulating currents. I hope you checked my calculations at each stage, I used a calculator which allows me to do complex arithmetic in either polar or rectangular form. If you had to convert each time it probably took you more lines of working. The important thing is to be able to see how the 2 initial questions were obtained.

In order to tackle the problem using Thévenin's Theorem we must determine the open circuit voltage between A and B, and in order to do this we need to find the current I_1 with $I_2 = 0$. This gives us the mesh equation:

$$(26 - j\,5)\,I_1 = 30 - (21{\cdot}65 + j\,12{\cdot}5) = 8{\cdot}35 - j\,12{\cdot}5$$

$$I_1 = 0{\cdot}399 - j\,0{\cdot}404 = 0{\cdot}568\angle{-45{\cdot}4°}$$

We may now find V_{oc}:

$$V_{oc} = 30 - I_1 \times (8 - j\,15)$$

$$V_{oc} = 30 - (-2{\cdot}87 - j\,9{\cdot}2)$$

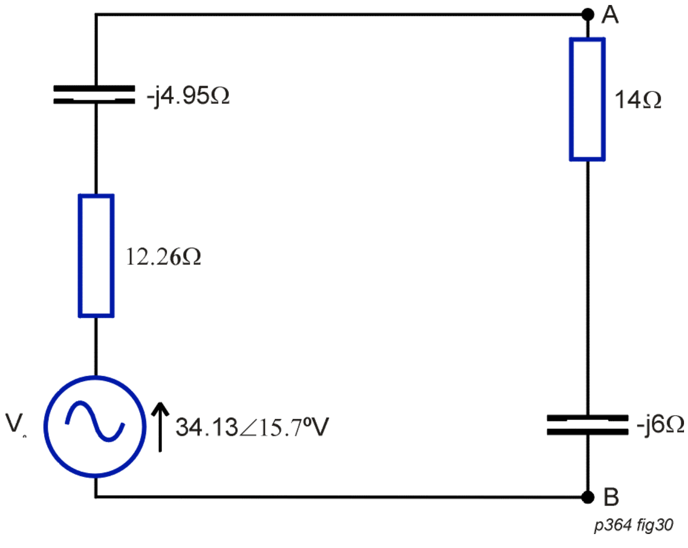
$$V_{oc} = 32{\cdot}87 + j\,9{\cdot}2 = 34{\cdot}13\angle 15{\cdot}7°\ \text{V}$$

The source impedance Z_o is the result of 8 – j15 in parallel with 18 + j10.

$$Z_0 = \frac{(8 - j15)\times(18 + j10)}{(8 - j15) + (18 + j10)}$$

$$Z_0 = 12{\cdot}26 - j\,4{\cdot}95$$

Hence the Thévenin equivalent circuit is:



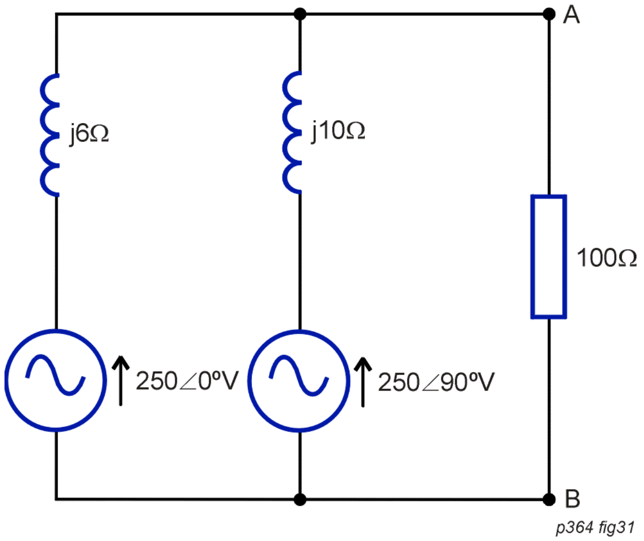
Hence $\quad I_{AB} = \dfrac{V_0}{Z_0 + Z_{AB}} = \dfrac{34\cdot 13\angle 15\cdot 7°}{(12\cdot 26 - j4\cdot 95) + (14 - j6)}$

$$I_{AB} = 1{\cdot}2\angle 38{\cdot}3°\ \text{A}$$

This is the same value of current as determined before.

SAQ 5-3-11

This SAQ is a simpler 2 mesh circuit which is very similar to the above example which I suggest you work out by both methods. The circuit is that of a Foster-Seeley FM discriminator at centre frequency and you are required to determine the voltage across the 100Ω load resistor.



Phasors

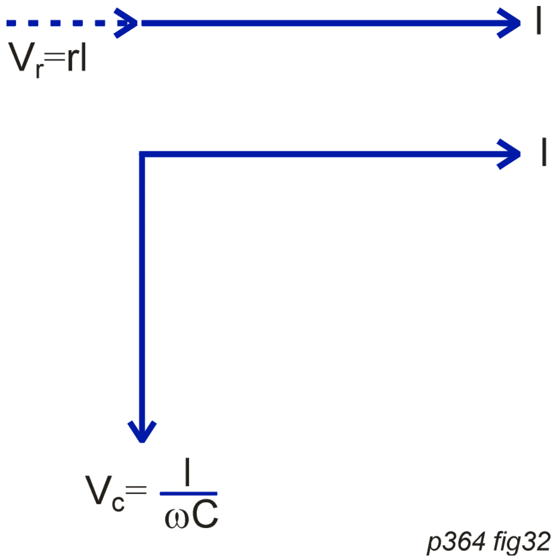
In the examples in this chapter we have used complex arithmetic to work out the required answers. This is in most cases the simplest and quickest way but it is sometimes useful to look at the phasor diagrams of the voltages and currents to get a better understanding.

For a resistor the voltage and current are in phase and hence the 2 phasors to represent them must be drawn in the same direction.

The capacitor the voltage will lag the current by 90°. Putting it the other way the current will lead the voltage by 90°. This is because the impedance is:

$$Z_C = \frac{-j}{\omega C}$$

The −j indicating an anti-clockwise rotation of 90°.

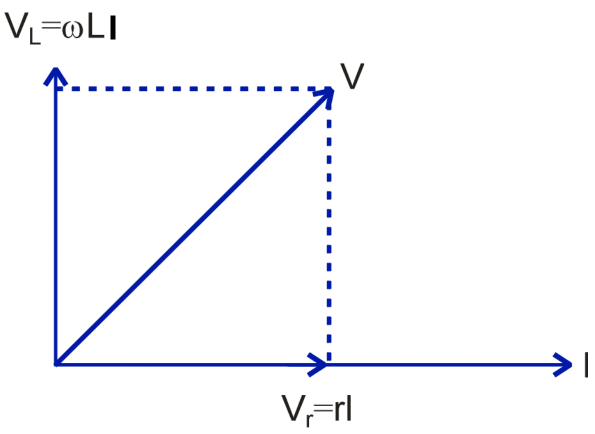


p364 fig32

In the case of a pure inductance since $Z_L = j\omega L$ the +j will result in a clockwise 90° turn of the voltage vector i.e. it would be in the opposite direction to the voltage across the capacitor shown above.

A mnemomic to assist you to remember which leads which in which case is the word CIVIL. The interpretation is in a Capacitor the current (I) leads (i.e. comes before) the Voltage (V). The Voltage (V) leads (comes before) the current (I) in an inductor (L).

For a coil which has both inductance and resistance in series the voltage across the circuit will be the sum of the 2 voltages which are at right angles to each other. Hence the total voltage will lead by an angle of less than 90°.



p364 fig33

Chapter 4

Resonance and Networks

Resonance

In the previous chapter we were looking at a type of analysis in which values were given and the unknowns determined using the rules of complex arithmetic. We are now going to look at the situation where the quantities in the circuit are given as variables and we are required to determine algebraic expressions for the required unknown in terms of the given variables. We will also be looking at problems which use the derived expressions with numeric values.

We will look at a number of different types of network but probably the most important ones are those which show a property called resonance. On your course at The Royal School of Signals you will be looking at the circuits in greater detail but this part of the distance learning package will give a good introduction.

Resonance is defined by the following statement. **A 2 terminal network containing reactive elements is said to be resonant when the current drawn is in phase with the applied voltage.**

The above definition means that if a circuit is resonant then the phase angle of the impedance is zero, hence the reactance is zero and the susceptance is zero. If any one of the 3 statements is true then the other 2 must also be true and therefore any one may be used as a test for resonance.

The frequency at which resonance occurs is called the *resonant frequency*. A circuit containing 2 reactive elements of opposite sign (i.e. an inductor and a capacitor) will usually have one resonant frequency which is denoted by f_o or ω_o. Circuits containing more than 2 reactive elements may well have a number of resonant frequencies. In this study we will only be concerned with circuits having one resonant frequency.

A circuit which exhibits resonance is called a *resonant circuit*. Such a circuit will frequently have either a maximum or a minimum of impedance at a frequency at or very near the resonant frequency and part of our analysis will be to determine these conditions.

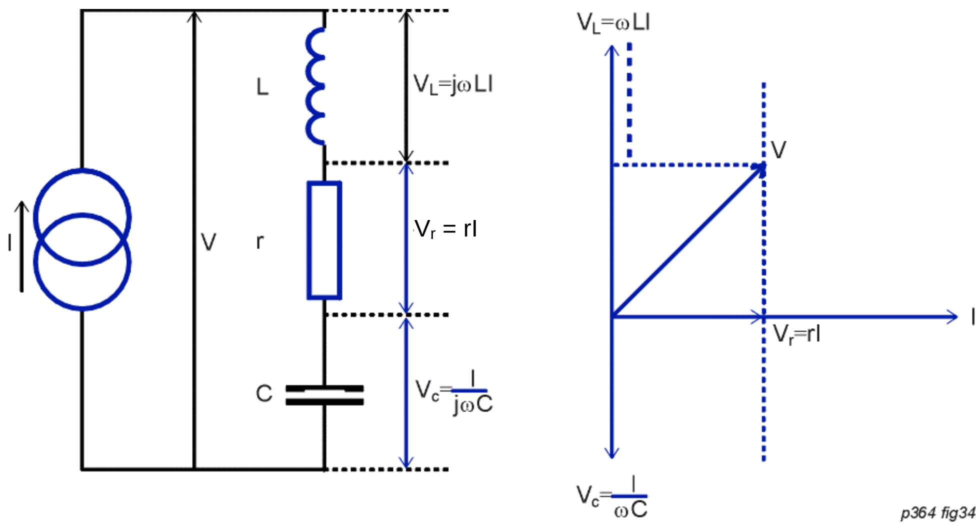
Series Resonance

The simplest resonant circuit comprises a resistor, an inductor and a capacitor in series. The resistor and inductor are in most practical circuits due to a coil, however in some cases additional resistance may be added.

The circuit shown on the next page comprising a coil, of resistance r and inductance L, in series with a capacitor of capacitance C; will have a resonant frequency at which the reactances of the 2 components will be equal in magnitude and the impedance will be a minimum equal to r.

For a constant voltage across the circuit the current will be a maximum and for a constant current through the circuit the voltage will be a minimum.

The diagram shows the circuit driven from a constant current source of current *1*A. Note the symbol for a constant current source which is one you may not have met before.



In the circuit diagram the expressions for the voltages across the components are given. The voltage across the coil is $V_r + V_L = (r + j\omega L)I$.

The total voltage across the circuit is given by:

$$V = V_r + V_L + V_C$$

$$V = \left(r + j\omega L + \frac{1}{j\omega C} \right) I$$

$$V = \left(r + j\left(\omega L - \frac{1}{\omega C} \right) \right) I$$

The impedance of the whole circuit is given by:

$$Z = r + j\left(\omega L - \frac{1}{\omega C} \right)$$

By inspection of the above expressions it can be seen that the previous statements concerning the properties of a series resonant circuit are true and that resonance occurs at the frequency at which the reactance of the circuit is zero, i.e. when:

$$\left(\omega L - \frac{1}{\omega C} \right) = 0$$

$\therefore$ At resonance, $\omega L = \frac{1}{\omega C}$

This gives $\omega_0 = \frac{1}{\sqrt{LC}}$

Where ω_0 is the angular resonant frequency in rad/sec i.e. $\omega_0 = 2\pi f_o$

and resonant frequency $$f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

The phasor diagram shows the condition of the circuit at some frequency above the resonant frequency. The voltage across the resistor V_r is shown in blue in phase with the current I. The voltage across the inductor V_L is shown in green leading the current by 90°, the current lags the voltage in an inductor. V_C is shown in red lagging the current by 90°.

The total voltage across the circuit shown as V leads the current by some angle and is found by adding V_L, V_C and V_r. The diagram shows V_C as a red dotted line being added to V_L and then V_r being added as a blue dotted line.

If the frequency is reduced then V_L will decrease (I and ω are constant), V_C will increase and V_r will remain constant. This will mean that V will move down along the black dotted line. When V is in phase with I (i.e. resonance) then v is a minimum.

At resonance in most practical cases V_L and V_C are much larger than V_r which is equal to V. This has not been shown in the diagram as it would be awkward to draw V_r large enough to see and keep the other 2 voltages on the paper. The ratio of either V_L or V_C to V is called the magnification factor and has the symbol Q. It is sometimes indicated as Q_0 as it is a value at resonance.

$$Q = \frac{V_L}{V} = \frac{\omega LI}{rI} = \frac{\omega L}{r}$$

$$Q = \frac{V_C}{V} = \frac{1}{\omega C\, rI} = \frac{1}{\omega Cr}$$

Note Q is the ratio of X over r as was stated on page 3-1. The reason the letter Q is used is that it is a measure of the Quality of the coil. It is also sometimes called the selectivity of the circuit. The reason for this will become apparent shortly.

As is seen from the phasor diagram and from the expression for Z, above resonance the impedance is greater than r and the circuit is inductive, below resonance it is capacitive.

Resonance or tuned circuits as they are frequently called are used in radio receivers to select the required signal, in oscillators to determine the frequency of oscillation and as components in filters and other circuits.

As a measure of how selective a circuit is, the bandwidth of the circuit is used. This is the range of frequencies (Δf) over which, in the series resonance case the modulus of the impedance has not increased above $\sqrt{2} \times r$. Equating this value to the expression for the modulus of the impedance will yield 2 frequencies which are known as the upper and lower 3 dB frequencies. The difference between these 2 frequencies is the 3 dB bandwidth, usually just called the bandwidth.

At the 3 dB frequencies if the circuit is driven from a constant voltage source the current will have fallen to 0·707 × its maximum value and the power will have fallen to ½ its maximum value. A fall of power to ½ is –3 dB.

It can be shown that:

$$\Delta f = \frac{f_0}{Q_0}$$

This is why Q is sometimes referred to as the selectivity. The two 3 dB frequencies f_1 and f_2 are not equally spaced about f_0.

SAQ 5-4-1

A series circuit is driven from a variable frequency oscillator with an output of 1V. At resonance the voltage across the 1 nF capacitor is 30V. The 3 dB bandwidth of the circuit is 2 kHz. Determine:

a. The Q of the circuit.

b. The resonant frequency.

c. The inductance of the circuit.

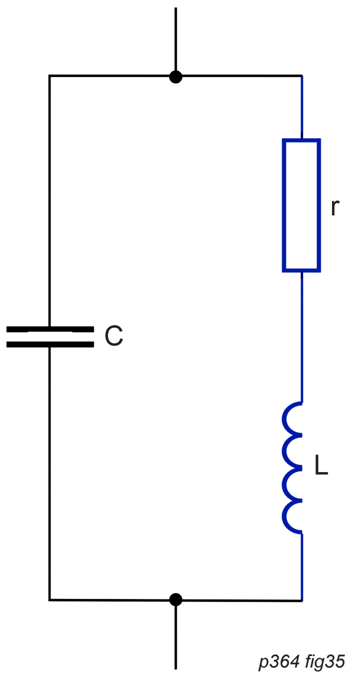
d. The resistance at resonance.

SAQ 5-4-2

Obtain an expression for Q_0 in terms of *r, L* and *C*.

Parallel Resonance

The simplest and most frequently met parallel resonant circuit is one comprising a coil, of inductance *L* and series resistance *r*, in parallel with a pure capacitor of capacitance *C*. This circuit is shown below:



p364 fig35

The best way of analysing parallel circuits is to use admittance. For the right hand branch the admittance is:

$$Y_L = \frac{1}{r + j\omega L}$$

$$Y_L = \frac{r}{r^2 + \omega^2 L^2} - \frac{j\omega L}{r^2 + \omega^2 L^2}$$

This is as on page 3-3.

For the left hand branch:

$$Y_C = j\omega C$$

Hence the total admittance is $Y = \frac{r}{r^2 + \omega^2 L^2} + j\left[\omega C - \frac{j\omega L}{r^2 + \omega^2 L^2}\right]$

SAQ 5-4-3

Show that, in the parallel circuit above, for resonance to occur:

$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{r^2}{L^2}}$$

Also show that at resonance:

$$Z = \frac{L}{Cr}$$

Note this resonant frequency is not the frequency at which the inductive and capacitive reactances are of equal magnitude, nor is it the frequency of maximum impedance. The maximum impedance is slightly greater than R_d and is capacitive. You will be learning more about parallel resonant circuits on your course.

Below resonance the parallel circuit is inductive and above resonance it is capacitive.

Most practical circuits are made with either the L or C adjustable over a limited range and therefore from a design point of view the simpler formula as used for series resonance is quite adequate. It actually gives a frequency nearer the frequency of maximum impedance which is frequently the required condition. It is simple to determine a relationship between the frequency of equal magnitude of reactances and the resonant frequency.

Let $\omega_1 = \frac{1}{\sqrt{LC}}$ (simplified resonance formula)

From $\omega_0 = \sqrt{\frac{1}{LC} - \frac{r^2}{L^2}}$

By re-arranging $\omega_0 = \frac{1}{\sqrt{LC}}\sqrt{1 - \frac{r^2 C}{L}}$

But $\frac{r^2 C}{L} = \frac{1}{Q^2}$ (see expressions for Q on page 4-3)

$\therefore$ $\omega_0 = \omega_1 \sqrt{1 - \frac{1}{Q^2}}$

This shows that if *Q* is 10 then the calculation based on the simple formula will only be ½% out! In the majority of cases the *Q* of the circuit is much greater than this.

Determining the actual 3 dB frequencies and the bandwidth for a parallel resonant circuit is complicated but the relationship given for the series circuit may be used with very little error unless the *Q* is very low.

$$\Delta f = \frac{f_0}{Q_0}$$

SAQ 5-4-4

A parallel circuit is resonant at a frequency of 16 kHz and has a bandwidth of 320 Hz. If the value of the capacitor used is 10 nF, determine:

a. The *Q* of the circuit.

b. The inductance of the coil.

c. The resistance of the coil.

d. The impedance of the circuit at resonance.

SAQ 5-4-5

A parallel resonant circuit is found to have a *Q* of 30 and a dynamic resistance at resonance of 9 kΩ. The coil in the circuit has an inductance of 9 mH. Determine:

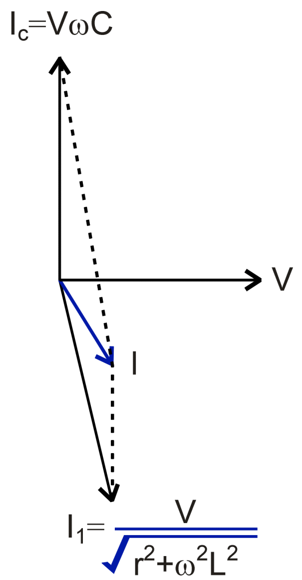
a. The resistance of the coil.

b. The capacitance of the capacitor.

c. The resonant frequency.

d. The 3 dB bandwidth.

To conclude the section dealing with parallel resonance I have shown a phasor diagram for the parallel circuit we have been considering. The applied voltage is the reference and the total current in black is then the sum of the currents in the 2 branches. The current in the capacitor is shown in red and the current in the coil which lags by slightly less than 90° is shown in green.



p364 fig36

SAQ 5-4-6

Is the phasor diagram drawn for a frequency above or below resonance?

Networks

The aim of this chapter is to look at the analysis of various networks of which the resonance networks which we have just considered are an important example. We will now look at other networks.

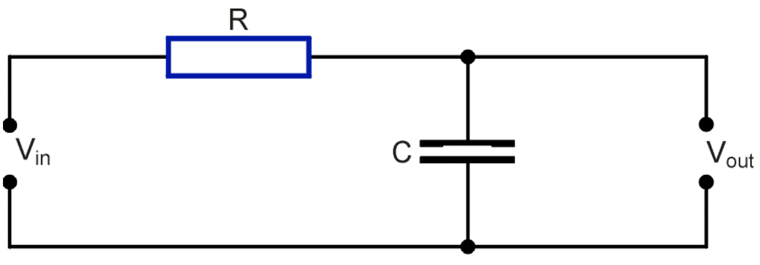
Phase Shift Networks

There are various applications where it is necessary to change the phase of a sine wave and this is done by means of *phase shift networks*. Various circuits could be used but it is normal to use one consisting of capacitors and resistors. Inductors are seldom used as they are more expensive than capacitors and tend to be more bulky. they also cannot be made pure, they always have a significant series resistance.

We will look at a number of simple *RC* networks where we are concerned with determining the ratio of the output voltage (V_{out}) to the input voltage (V_{in}) as a complex function of the real variable ω.

That paragraph probably needs some explanation. What we are trying to determine is an expression which will involve *R*, *C* and ω which equates to the ratio V_{out}/V_{in}. If values are put in the expression the result will be a complex operator giving the relationship between V_{out} and V_{in}. *R* and *C* are considered to be constants which is why I have said it is a function of ω.

The above will become clearer as we do a simple example. Consider a simple series circuit of one resistor and one capacitor with the output across the capacitor as shown in the diagram below.



We always assume that the circuit is not loaded (i.e. the output terminals are open circuit). If there is a load given then make it part of the circuit.

The voltages may now be expressed in terms of the current through *R* and *C*.

$$V_{\text{in}} = \left(R - \text{j}\frac{1}{\omega C} \right) I$$

$$V_{\text{out}} = -\text{j}\frac{1}{\omega C} I$$

Dividing one expression by the other and cancelling *I* quickly leads to:

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{1}{1 + \text{j}\omega CR}$$

In modulus and angle form this gives $\dfrac{1}{\sqrt{1+\omega^2 C^2 R^2}} \angle -\tan^{-1}(\omega CR)$

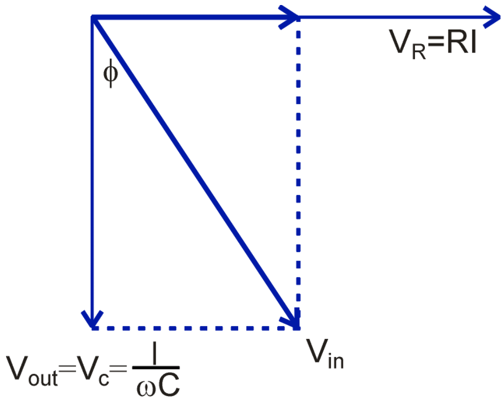
This relationship is known as the *transfer function* of the network as a function of ω. On your course you will meet other ways of expressing the transfer function.

This shows that the output lags the voltage by $\tan^{-1}$ (ω*CR*). The circuit is called a phase retarding circuit. It also shows that as the frequency is increased the magnitude of the output is reduced. Hence the circuit may be described as a low pass filter. The 3 dB point on the response is given by:

$$\omega = \frac{1}{CR}$$

At this frequency the phase change is $\tan^{-1} 1 = 45°$.

The phasor diagram below shows the relationship between the voltages and the current.



V_{out} lags V_{in} by ϕ where

$$\tan\phi = \frac{RI}{\frac{I}{\omega C}}$$

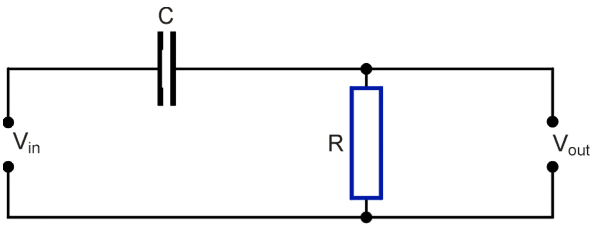
$$= \omega CR$$

$$\phi = \tan^{-1}(\omega CR)$$

p364 fig38

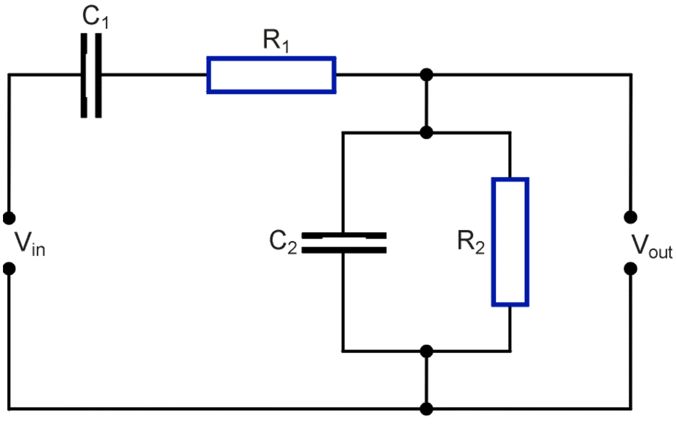
SAQ 5-4-7

Find the phase angle between V_{out} and V_{in}, stating whether the network is advancing or retarding the phase. (Hint: draw the phasor diagram).



p364 fig39

The network shown below is widely used in tuneable oscillators and also to produce what are called notch filters which select or reject a narrow band of frequencies. I will start the analysis and leave you to finish it as SAQ 5-4-8.



For the parallel components:

$$Z_p = \frac{\text{Product}}{\text{Sum}} = \frac{R_2 \times \frac{1}{j\omega C_2}}{R_2 + \frac{1}{j\omega C_2}}$$

This simplifies to:

$$Z_p = \frac{R_2}{1 + j\omega C_2 R_2}$$

$$V_{out} = I \times Z_p$$

The impedance of the series part of the circuit is:

$$Z_s = R_1 + \frac{1}{j\omega C_1} = \frac{1 + j\omega C_1 R_1}{j\omega C_1}$$

The total impedance is given by:

$$Z_T = Z_s + Z_p = \frac{1 + j\omega C_1 R_1}{j\omega C_1} + \frac{R_2}{1 + j\omega C_2 R_2}$$

$$Z_T = \frac{(1 + j\omega C_1 R_1)(1 + j\omega C_2 R_2) + j\omega C_1 R_2}{j\omega C_1 (1 + j\omega C_2 R_2)}$$

$$V_{in} = I \times Z_T$$

SAQ 5-4-8

Using the above expressions as a starting point determine an expression for the transfer function and hence or otherwise determine an expression for the value of ω to give V_{out} in phase with V_{in}.

At this frequency determine the value of the transfer function if $R_1 = R_2$ and $C_1 = C_2$.

There are other situations which may need to be considered and require the use of complex algebra. The SAQ below is your final SAQ and the end of the section.

I hope you have worked through all the SAQs and will be fully prepared for the examination.

SAQ 5-4-9

A series circuit consisting of a coil, of inductance 5 mH and resistance 200Ω, in series with a loss free capacitor of 100 nF capacitance is fed from a constant current AC supply. The voltages across the coil (involving resistance and inductance) and the capacitor are displayed on a cathode ray oscilloscope. The frequency is varied until the 2 displays are of equal amplitude:

a. What is this frequency?

b. What is the phase angle between these voltages at this frequency?

Chapter 5

Solutions to SAQs

Solutions to SAQs

SAQ 5-1-1

To simplify the circuit the first thing to do is to add the 2Ω and 4Ω resistors to give one series resistor of 6Ω.

This effective 6Ω resistor is now in parallel with the 3Ω resistor, combining these using the standard formula gives:

$$R = \frac{6 \times 3}{6 + 3} = 2\Omega$$

The total voltage around the simplified circuit in a clockwise direction is 8V minus 3V which is 5V.

The total resistance is 8 + 2 = 10Ω

The total current in the circuit is thus:

$$I = \frac{V}{R} = \frac{5}{10} = 0 \cdot 5\,\text{A}$$

This is the current in the 8Ω resistor and also flows in the effective total resistance of the other 3 resistors which is 2Ω as calculated above.

The voltage across this effective resistance is therefore:

$$V = I\,R = 0{\cdot}5 \times 2 = 1\text{V}$$

This is the voltage across both the 3Ω resistor and the effective 6Ω of the sum of the 2Ω and 4Ω resistors. Hence the current in the 3Ω resistor is 1/3 A and through the 2Ω and 4Ω resistors is 1/6 A.

SAQ 5-1-2

This SAQ is very similar to the previous one and I have therefore only given the answers.

Current in 30Ω = 0·166A, 6Ω = 0·833A, 10Ω = 0·8A, 20Ω = 0·2A.

The same current flows in both 20Ω resistors.

SAQ 5-1-3

The fourth resistor has a resistance of 4Ω.

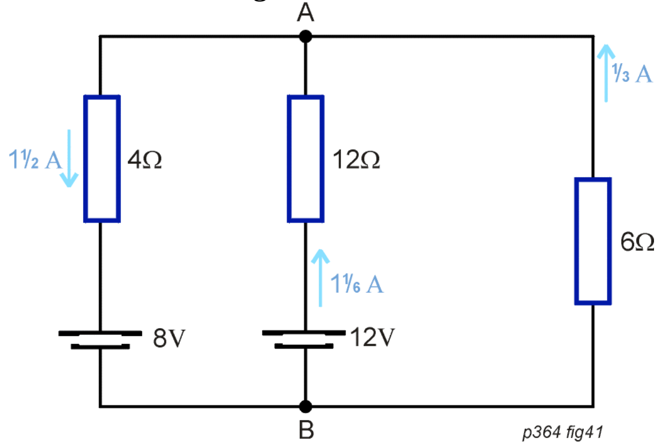
The total power in the 3 resistors is 96W.

SAQ 5-1-4

R_a = 125Ω R_b = 250 Ω

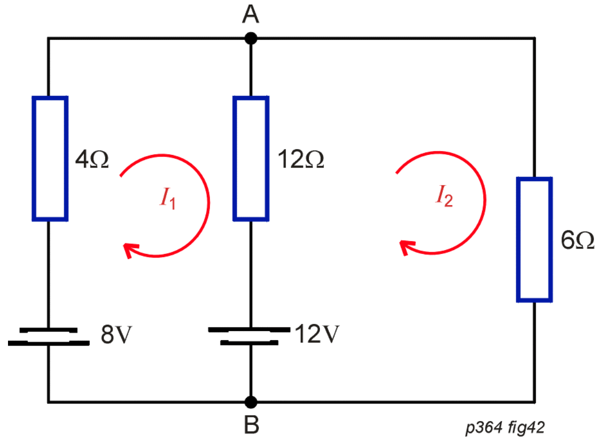
SAQ 5-1-5

The currents are as shown in the diagram.



SAQ 5-1-6

The currents are of course the same as those shown above. The analysis is shown below.



The mesh equations are:

$$16\,I_1 - 12\,I_2 = -20$$

$$-12\,I_1 + 18\,I_2 = 12$$

These solve to give:

$$I_1 = -1\tfrac{1}{2}\text{ A} \quad I_2 = -\tfrac{1}{3}\text{ A}$$

Hence the currents in the branches may be found.

SAQ 5-1-7

i. Neither; the path is not closed.

ii. Loop; closed path encloses another loop hence not a mesh.

iii. Mesh.

iv. Mesh.

SAQ 5-1-8

The required mesh equations are:

$$98\ I_1 - 15\ I_2 - 56\ I_3 = 8$$

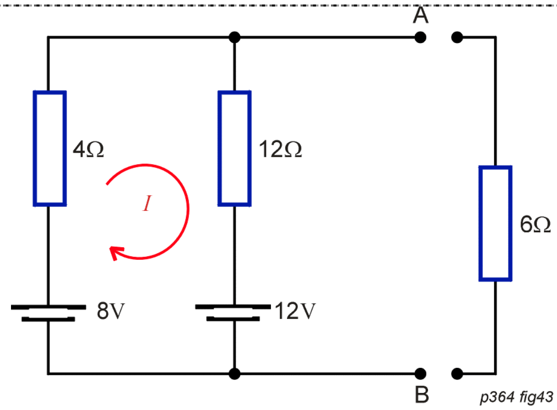
$$-15\ I_1 + 95\ I_2 - 33\ I_3 = -29$$

$$56\ I_1 - 33\ I_2 + 109\ I_3 = -6$$

The 3 mesh currents are: −116·5 mA; −406·3 mA; −237·9 mA.

The minus signs indicate that all the currents are anti-clockwise.

SAQ 5-1-9



In this case the circulating current in the left hand loop, taking clockwise as positive, is given by:

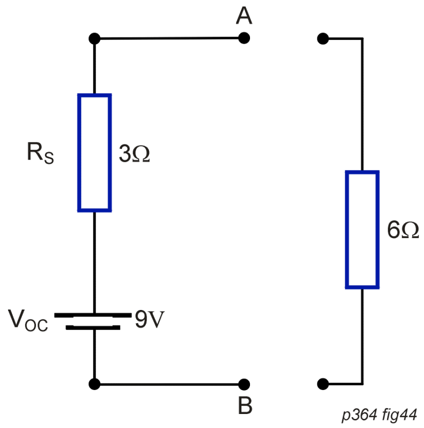
$$I = \frac{8-12}{4+12} = -¼\ \text{A}$$

Hence: $V_{oc} = 8 - (-¼ \times 4) = 9V$

Or $V_{oc} = 12 + (-¼ \times 12) = 9V$

The source resistance is the same as before 4//12 = 3Ω

Hence the circuit becomes:



Hence the current in the 6Ω resistor is given by:

$$i = \frac{9}{3+6} = 1\text{A}$$

SAQ 5-1-10

The working for this problem is very similar to that for the example worked in the text.

Taking A as reference earth gives:

$$V_{\mathrm{B}} = 6 \times \frac{25}{60} = 2 \cdot 5\,\mathrm{V}$$

$$V_{\mathrm{D}} = 10 \times \frac{45}{65} = 6 \cdot 923\,\mathrm{V}$$

Did you miss the fact that the polarity of the 6V battery has been reversed?

$$R_{\mathrm{S}} = 25//35 \ + \ 45//20$$

$$R_{\mathrm{s}} = \frac{25 \times 35}{25 + 35} + \frac{45 \times 20}{45 + 20} = 28 \cdot 43\Omega$$

Hence current from D to B is given by:

$$i = \frac{6 \cdot 923 - 2 \cdot 5}{28 \cdot 43 + 15} = 101 \cdot 8\,\mathrm{mA}$$

SAQ 5-2-1

i. The period is the reciprocal of the frequency hence:

Period T = 250 μs

You should have given the answer as 250 μs not 0·25 ms.

ii. The rms value is the peak value divided by √2.

V = 3·54V

iii. As the value is positive and decreasing, the angle must be between 90° and 180°.

Phase angle = 180° – arcsin 5/2 = 180° – 23·58° = 156·42°

The mathematical expression for the waveform is:

$$v = \hat{V} \sin\left(\omega t + 156 \cdot 4°\right)$$

$$v = \hat{V} \sin\left(1{\cdot}44 \times 10^{6} \times t + 156 \cdot 4°\right)$$

SAQ 5-2-2

$\omega = 2 \times \pi \times 5 \times 10^3 = 31{\cdot}4$ rad/s

at $t = 25$ µs $\omega t = 0{\cdot}785$ rad

$v_1 = 7 \times \sin 0{\cdot}785 = 4{\cdot}95$V remember to set your calculator to radians.

Alternatively:

Period $T = 1/f = 200$ µs

The angle corresponding to 25 µs is $360° \times \frac{25}{200} = 45°$

Hence $v_1 = 7 \times \sin 45° = 4{\cdot}95$V

For the second wave $v_2 = 7 \times \sin (45 + 30)° = 6{\cdot}76$V (using degrees)

Or $v_2 = 7 \times \sin (\cdot 785 + \cdot 524) = 6{\cdot}76$V (using radians)

$30° = \pi/6 = 0{\cdot}524$ rad

To convert from degrees to radians multiply by $\frac{\pi}{180}$.

To convert radians to degrees multiply by $\frac{180}{\pi} = 57{\cdot}3$.

SAQ 5-2-3

$v_a = 200 \sin \omega t = 200\angle 0° = 200 + \text{j}\ 0$

$v_b = 170 \sin (\omega t - \pi/4) = 170\angle -\pi/4 = 170\angle -45° = 120{\cdot}2 - \text{j}\ 120{\cdot}2$

$v_c = 230 \sin (\omega t - \pi/6) = 230\angle -\pi/6 = 230\angle 30° = 199{\cdot}2 + \text{j}\ 115$

$v_a + v_b + v_c = 519{\cdot}4 - \text{j}\ 5{\cdot}2 = 519{\cdot}4\angle -0{\cdot}57° = 519{\cdot}4\angle -0{\cdot}01$

The total voltage is $519{\cdot}4 \sin (\omega t + 0{\cdot}01)$

Note ϕ is in radians as it was given in the question.

SAQ 5-3-1

At $f = 2$ kHz

$X = 2 \times \pi \times 2 \times 10^3 \times 600 \times 10^{-6}$

$X = 7{\cdot}54\Omega$ $Z = 14 + \text{j}\ 7{\cdot}54 = 15{\cdot}9\angle 28{\cdot}3°$

The current would be $I = \frac{10}{15\cdot 9\angle 28\cdot 3°} = 629$ mA lagging the voltage by 28·3°.

At f = 10 kHz

$$X = 37{\cdot}7\Omega \qquad Z = 14 + \mathrm{j}\,37{\cdot}7 = 40{\cdot}2\angle 69{\cdot}6°$$

$$I = \frac{10}{40{\cdot}2\angle 69{\cdot}6°} = 248{\cdot}7\angle -69{\cdot}6 \text{ mA}$$

At ω = 100 krad/s

$$X = 60\Omega \qquad Z = 14 + \mathrm{j}\,60 = 61{\cdot}6\angle 76{\cdot}9°$$

$$I = \frac{10}{61{\cdot}6\angle 76{\cdot}9°} = 162\angle -76{\cdot}9° \text{ mA}$$

SAQ 5-3-2

At 10 kHz the reactance is – 159Ω, the impedance is – j 159Ω or 159∠–90°

At 250 kHz the reactance is –6·37Ω, the impedance is –j 6·37Ω or 6·37∠–90°Ω

Note the distinction between reactance and impedance.

SAQ 5-3-3

$$Z_1 = 47\angle 35° = 38{\cdot}5 + \mathrm{j}\,27{\cdot}0$$

$$Z_2 = 32 + \mathrm{j}\,17 = 36{\cdot}24\angle 28°$$

a. In series $Z = Z_1 + Z_2 = 38{\cdot}5 + \mathrm{j}\,27{\cdot}0 + 32 + \mathrm{j}\,17 = 70{\cdot}5 + \mathrm{j}\,44$

$$Z = 83{\cdot}1\angle 32°$$

b. In parallel $Z = \dfrac{Z_1 \times Z_2}{Z_1 + Z_2}$

$$Z = \frac{47\angle 35° \times 36\cdot 24\angle 28°}{83{\cdot}1\angle 2°}$$

$$Z = 20{\cdot}5\angle 31° = 17\cdot 57 + \mathrm{j}10\cdot 56$$

SAQ 5-3-4

$$Z = 15 - \mathrm{j}20 = 25\angle -53{\cdot}1°\Omega$$

$$Y = \frac{1}{Z} = \frac{1}{25\angle -53\cdot 1°} = 0{\cdot}04\angle 53{\cdot}1°$$

$$Y = 40\angle 53{\cdot}1° = 24 + \mathrm{j}32 \text{ mS}$$

$$\therefore G = 24 \text{ mS} \quad \text{and} \quad B = 32 \text{ mS}$$

Note Y, G and B are all in siemens.

Alternatively after the first line above:

$$G = \frac{R}{|Z|^2} = \frac{15}{25^2} = 24 \text{ mS}$$

and $$B = \frac{-X}{|Z|^2} = \frac{-(-20)}{25^2} = 32 \text{ mS}$$

Hence Y = 24 + j 32 = 40∠53·1° mS

SAQ 5-3-5

In this case we are given

$|Z|$ = 85Ω R= 15Ω f = 25 kHz

We know $|Z|^2 = R^2 + X^2$

Hence $X = \sqrt{(|Z|^2 - R^{23})} = \sqrt{(85^2 - 15^2)} = \sqrt{7000}$

$X = \pm 83{\cdot}7\Omega$

X is positive if the circuit is inductive and negative if the circuit is capacitive and there is no way of determining the sign from the information given in the question.

If x = + 83·7Ω then $$L = \frac{83{\cdot}7}{2 \times \pi \times 25 \times 10^3} = 533\,\mu\text{H}$$

iF x = −83·7Ω then $$C = \frac{1}{2 \times \pi \times 25 \times 10^3 \times 83{\cdot}7} = 76 \text{ nF}$$

SAQ 5-3-6

In this case we are given:

G = 30 mS and X = 16 Ω

and we have to determine possible values for R and B.

It is not obvious how we may proceed, we know that simply taking reciprocals does **not** give us the answer. However if we write down the relationships that we do know we may see how to proceed.

$$|Z|^2 = R^2 + X^2 \quad G = \frac{R}{|Z|^2} \quad \text{and} \quad B = \frac{-X}{|Z|^2}$$

This doesn't look as though it is getting us far at first sight but if we substitute for R in the first equation using the second we will obtain an expression in which $|Z|^2$ is the only unknown.

$$|Z|^2 = (G \times |Z|^2)^2 + X^2$$

$$|Z|^2 = G^2 \times |Z|^4 + X^2$$

Putting in the numerical values which we know gives:

$$|Z|^2 = 0{\cdot}0009 \times |Z|^4 + 256$$

This is a quadratic equation in $|Z|^2$. Re-arranging gives:

$$0{\cdot}0009 \times |Z|^4 - |Z|^2 + 256 = 0$$

Hence $$|Z|^2 = \frac{1 \pm \sqrt{1 - 4 \times 9 \times 10^{-4} \times 256}}{2 \times 9 \times 10^{-4}}$$

$$|Z|^2 = 711{\cdot}11 \text{ or } 400$$

$$|Z| = 26{\cdot}67 \text{ or } 20$$

If $|Z| = 20$ then: $Z = 20\angle 53{\cdot}13° = 12 + \text{j } 16\Omega$

$Y = 50\angle -53{\cdot}13° = 30 - \text{j } 40\Omega$

If $|Z| = 26{\cdot}67$ then: $Z = 26{\cdot}67\angle 36{\cdot}87° = 21{\cdot}3 + \text{j } 16\Omega$

$Y = 37{\cdot}5\angle -36{\cdot}87° = 30 - \text{j } 22{\cdot}5\Omega$

SAQ 5-3-7

The current through the bulb and hence through the whole circuit I is given by:

$$I = \frac{W}{V} = \frac{60}{100} = 0{\cdot}6\,\text{A}$$

For case a.

Total resistance $$R_\text{T} = \frac{V}{I} = \frac{240}{0{\cdot}6} = 400\Omega$$

Resistance of lamp $$R_\text{L} = \frac{100}{0{\cdot}6} = 167\Omega$$

Extra non-inductive resistance required = $R_\text{T} - R_\text{L}$ = 233Ω

Total power drawn = $I^2R = 0{\cdot}6^2 \times 400$ = 144W

Alternative power = $IV = 0{\cdot}6 \times 240$ = 144W

b.

In this case it is the magnitude of the total impedance of the circuit which must be 400Ω. Hence as

$$|Z| = \sqrt{(R^2 + X^2)}$$

$$400 = \sqrt{(167^2 + X^2)}$$

$$X = 363{\cdot}5\Omega$$

But $X = \omega L$

$$L = \frac{365{\cdot}5}{2\times\pi\times50} = 1{\cdot}16\,\text{H}$$

In this case the power is again $I^2R = 0{\cdot}6^2 \times 167 = 60\text{W}$

Note that the power could be calculated from *VI* but it is the voltage across the resistance which must be used. Using an inductor to limit the current is obviously much more efficient than using a resistor. This is still true with a practical inductor which has resistance although there would then be some losses in the coil.

SAQ 5-3-8

The 3 voltages are:

$$V = 33{\cdot}3\angle 3{\cdot}5° \qquad V_1 = 27{\cdot}0\angle 68{\cdot}3° \qquad V_2 = 32{\cdot}7\angle -44{\cdot}9°$$

And the currents:

$$I_{100\Omega} = 100\text{ mA} \qquad I_{470\Omega} = 57{\cdot}5\angle 68{\cdot}3°\text{ mA}$$

$$I_{680\Omega} = 48{\cdot}1\angle -44{\cdot}9°\text{ mA} \qquad I_{30\text{ nF}} = 123{\cdot}4\angle 45{\cdot}1°\text{ mA}$$

If you have got it right, well done! If you haven't then try to find your error and correct it before you go through my solution, if you just follow my working you won't learn as much as if you puzzle it out for yourself.

In this example we have 2 reactive components and we will need to know their reactance, so it is as well to work this out first.

$$X_L = \omega L = 2 \times \pi \times 20 \times 10^3 \times 2 \times 10^{-3}$$

$$X_L = 251\Omega$$

$$X_C = \frac{-1}{2\pi fC} = \frac{-1}{2\times\pi\times20\times10^3\times30\times10^{-9}}$$

$$X_C = -265\Omega$$

We can now determine V_1.

The impedance of the branch containing the inductor is given by:

$$Z = 100 + \text{j}\ 251 = 270{\cdot}2\angle 68{\cdot}3°\Omega$$

$$V_1 = IZ = 0{\cdot}1 \times Z = 27{\cdot}0\angle 68{\cdot}3°\text{V}$$

$$V_1 = 10 + \text{j}\ 25{\cdot}1\text{V}$$

We can now find $I_{470\Omega}$:

$$I_{470\Omega} = \frac{V}{R} = \frac{27{\cdot}0\angle 68{\cdot}3°}{470}$$

$$I_{470\Omega} = 57{\cdot}5\angle 68{\cdot}3° = 21{\cdot}3 + \text{j}53{\cdot}4\ \text{mA}$$

The total current in the circuit, i.e. the current drawn from the source is given by:

$$I_\text{T} = 100 + I_{470\Omega} = 121{\cdot}3 + \text{j}\ 53{\cdot}4\ \text{mA}$$

$$I_\text{T} = 132{\cdot}5\angle 23{\cdot}8°\ \text{mA}$$

The impedance of the capacitor and shunt resistor may now be determined as the product divided by the sum.

$$Z_\text{C} = \frac{680 \times -\text{j}265}{680 - \text{j}265}$$

$$Z_\text{C} = \frac{680 \times 265\angle -90°}{729{\cdot}8\angle -21{\cdot}3°}$$

$$Z_\text{C} = 246{\cdot}9\angle -68{\cdot}7°\Omega$$

We now have: $V_2 = I_\text{T}Z_\text{C} = 0{\cdot}1325\angle 23{\cdot}8° \times 246{\cdot}9\angle -68{\cdot}7°\text{V}$

$V_2 = 32{\cdot}7\angle -44{\cdot}9° = 23{\cdot}2 - \text{j}\ 23{\cdot}1\text{V}$

Hence $V = V_1 + V_2 = 10 + \text{j}\ 25{\cdot}1 + 23{\cdot}2 - \text{j}\ 23{\cdot}1$

$V = 33{\cdot}2 + \text{j}\ 2 = 33{\cdot}3\angle 3{\cdot}5°\text{V}$

We still have 2 currents to determine but as we have V_2 this is simple.

$$I_{680\Omega} = \frac{32{\cdot}7\angle -44{\cdot}9°}{680\angle 0°} = 48{\cdot}1\angle -44{\cdot}9°\ \text{mA}$$

$$I_{30nF} = \frac{32{\cdot}7\angle -44{\cdot}9°}{265\angle -90°} = 123{\cdot}4\angle 45{\cdot}1°\ \text{mA}$$

SAQ 5-3-9 $R = 72{\cdot}5\Omega$, $f = 9{\cdot}98$ kHz, $V_T = 29{\cdot}15$V, $L = 1{\cdot}85$ mH and $V_{LC} = 15$V

SAQ 5-3-10 $I_T = 43{\cdot}4\angle 22{\cdot}35°$ $I_C = 37{\cdot}8\angle 51{\cdot}8°$ $P = 4{\cdot}01$W

SAQ 5-3-11 The voltage across the 100Ω resistor is $182\angle 28{\cdot}8°$

SAQ 5-4-1

a. The voltage across the circuit at resonance is Q times the voltage across the whole circuit hence $Q = 30$.

b. The bandwidth is the resonant frequency divided by Q. Hence:

$$f_0 = \Delta f \times Q$$

$$f_0 = 2 \times 30 = 60 \text{ kHz}$$

c.

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$\therefore \quad L = \frac{1}{4\pi^2 f_0^2 C}$$

$$L = \frac{1}{4\pi^2\left(60\times10^3\right)^2\times1\times10^{-9}}$$

$$L = 7{\cdot}04 \text{ mH}$$

d.

$$Q = \frac{1}{\omega Cr}$$

$$r = \frac{1}{\omega CQ} = \frac{1}{2\pi\times60\times10^3\times1\times10^{-9}\times30}$$

$$r = 88{\cdot}4\Omega$$

SAQ 5-4-2 By substituting the expression for ω_0 in either of the expressions for Q it is easy to obtain:

$$Q_0 = \frac{1}{r}\sqrt{\frac{L}{C}}$$

SAQ 5-4-3

One of the conditions which we stated for resonance was that the susceptance must be zero. The susceptance of this circuit is given by the term in brackets and for this to be zero:

$$\omega C = \frac{\omega L}{r^2 + \omega^2 L^2}$$

$$\therefore \quad C = \frac{L}{r^2 + {\omega_0}^2 L^2}$$

Multiplying both sides by $r^2 + \omega^2 L^2$ and dividing by C gives:

$$r^2 + \omega^2 L^2 = \frac{L}{C} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\therefore \quad {\omega_0}^2 L^2 = \frac{L}{C} - r^2$$

Dividing by L $\qquad {w_0}^2 = \frac{1}{LC} - \frac{r^2}{L^2}$

$$w_0 = \sqrt{\frac{1}{LC} - \frac{r^2}{L^2}}$$

At resonance the impedance will be purely real by definition and hence the admittance will be the first term in the expression in the main text with ω_0 substituting for ω. This gives:

$$Y = \frac{r}{r^2 + \omega_0^2 L^2}$$

Substituting for $r^2 + \omega_0^2 L^2$ using (1) above gives:

$$Y = \frac{Cr}{L}$$

Hence $\qquad Z = \frac{L}{Cr}$

This is called the dynamic resistance and is given the symbol R_d.

SAQ 5-4-4

a. $\qquad Q = \frac{16000}{320} = 50$

b. As the Q is >>10 we may use the approximate formula for the resonant frequency in order to determine L.

$$L = \frac{1}{4\pi^2 \times \left(16 \times 10^3\right)^2 \times 10 \times 10^{-9}} = 9{\cdot}9 \text{ mH}$$

If you work to 3 places of decimals the approx answer is 9·895 mH and the accurate answer comes to 9·891 mH.

c. There are various expressions which may be used to determine the resistance of the coil but this is the one I used.

$$Q = \frac{1}{\omega C r}$$

$$r = \frac{1}{\omega \mathrm{CQ}} = \frac{1}{2 \times \pi \times 16 \times 10^{3} \times 10 \times 10^{-9} \times 50}$$

$$r = 19{\cdot}9\,\Omega$$

d. $$R_{\mathrm{d}} = \frac{L}{Cr} = \frac{9{\cdot}9 \times 10^{-3}}{10 \times 10^{-9} \times 19{\cdot}9} = 49{\cdot}7\ \mathrm{k}\Omega$$

SAQ 5-4-5

This question is not as easy as the previous one as there is no single expression which we have dealt with in the text which will allow any of the required answers to be determined directly from the given information. However 2 expressions may be used to produce a new one which will give us our first answer.

a. By eliminating the ratio *L/C* from the 2 expressions:

$$R_{\mathrm{d}} = \frac{L}{Cr} \quad and \quad Q_0 = \frac{1}{r}\sqrt{\frac{L}{C}}$$

we obtain the expression $R_{\mathrm{d}} = rQ^2$

Using this expression $r = \frac{R_{\mathrm{d}}}{Q^2} = \frac{9000}{900} = 10\,\Omega$

b. $$R_{\mathrm{d}} = \frac{L}{Cr} \quad \therefore \quad C = \frac{L}{R_{\mathrm{d}} r} = \frac{0{\cdot}009}{9000 \times 10} = 100\ \mathrm{nF}$$

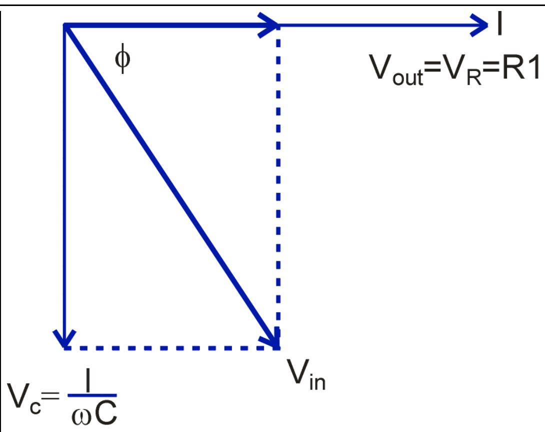
c. $$f_0 = \frac{1}{2\pi\sqrt{LC}} = 5{\cdot}3\ \mathrm{kHz}$$

d. $$\Delta f = \frac{f_0}{Q_0} = 177\ \mathrm{Hz}$$

SAQ 5-4-6

The frequency is below resonance, and, as shown, the inductive current dominates, making the overall circuit impedance inductive-resistive. Increasing frequency will increase I_C and decrease I_L, eventually bringing I into phase with V at resonance, and making the circuit impedance purely resistive.

SAQ 5-4-7



p364 fig45

In this case V_{out} is the same as V_R.

And tan ϕ = $\dfrac{\frac{I}{\omega C}}{RI}$

$$\phi = \tan^{-1}\frac{1}{\omega CR}$$

In this case V_{out} leads V_i

SAQ 5-4-8

a. After some re-arrangement you should have obtained:

$$\frac{V_{out}}{V_{in}} = \frac{\omega C_1 R_2}{\omega C_1\left(R_1 + R_2\right) + \omega C_2 R_2 + j\left(\omega^2 C_1 C_2 R_1 R_2 - 1\right)}$$

b. For the phase shift to be zero the transfer function must be real, which means that the j term in the denominator must be zero.

$$\omega^2 C_1 C_2 R_1 R_2 - 1 = 0$$

$$\omega = \frac{1}{\sqrt{C_1 C_2 R_1 R_2}}$$

c. With the j term zero ω cancels out and putting $R_1 = R_2$ and $C_1 = C_2$ will lead to a value for the transfer function of 1/3.

SAQ 5-4-9

For both the coil and the capacitor the magnitude or modulus of the voltage is the product of the current and the modulus of the impedance.

$$|V_1| = I\sqrt{200^2 + 5^2 \times 10^{-6} \times \omega^2}$$

$$|V_2| = \frac{1}{100 \times 10^{-9}\omega}$$

But $|V_1| = |V_2|$

∴ $(4 \times 10^4 + 25 \times 10^{-6}\,\omega^2)(10^{-14}\,\omega^2) = 1$

$$25 \times 10^{-20}\,\omega^4 + 4 \times 10^{-10}\,\omega^2 - 1 = 0$$

This may be solved for ω^2 using the standard formula as it is a quadratic in ω^2. We must take the positive value as it is a square.

$$\omega^2 = 1{\cdot}354 \times 10^9$$

ω = 36797 rad/s $\qquad$ f = 5·856 kHz

The angle between V_1 and V_2 will be the sum of the 90° that V_2 lags I and the angle that V_1 leads I.

$$\text{required angle if } 90° + \tan^{-1}\frac{\omega L}{r} = 90° + 42{\cdot}6° = 132{\cdot}6°$$